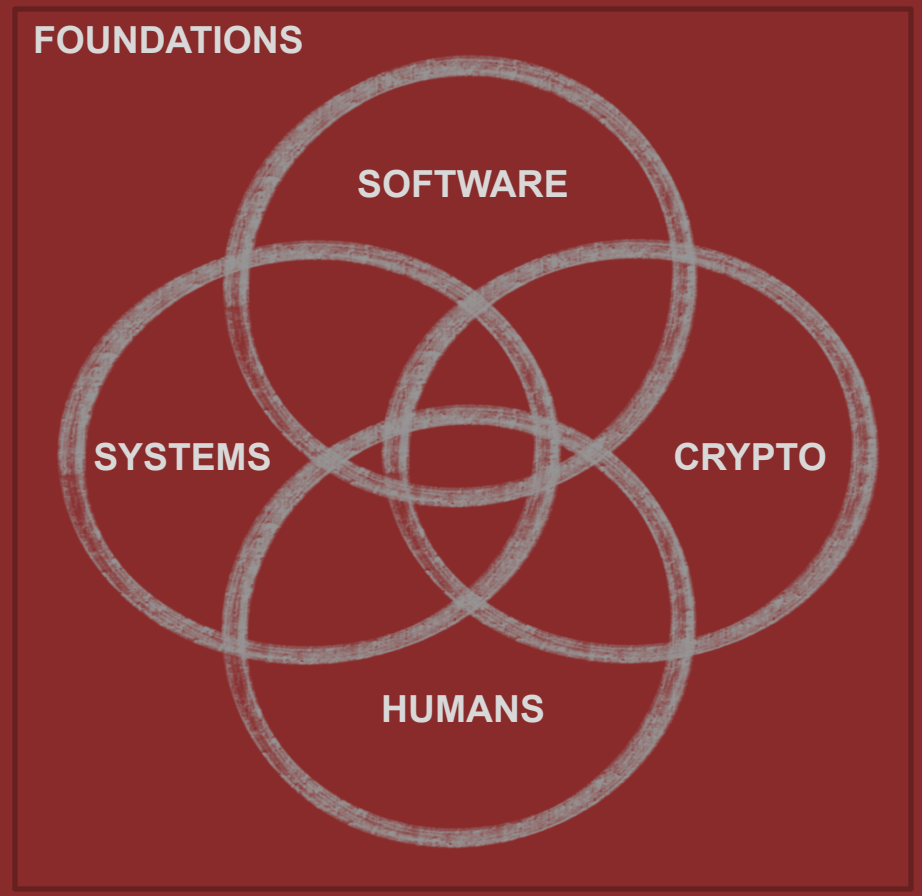
Διάλεξη #13 - Pseudorandom Functions & Permutations

Εθνικό και Καποδιστριακό Πανεπιστήμιο Αθηνών

Εισαγωγή στην Ασφάλεια

Θανάσης Αυγερινός



Huge thank you to David Brumley from Carnegie Mellon University for the guidance and content input while developing this class (some slides from Dan Boneh @ Stanford!)

Ανακοινώσεις / Διευκρινίσεις

- Την επόμενη εβδομάδα δεν έχουμε μάθημα Τετάρτη/Πέμπτη
  - Αναπλήρωση TBD
- Προθεσμία για την εργασία #2: 22 Μαΐου

Την προηγούμενη φορά

- Problems with just OTP
- Randomness and Pseudorandomness
- Probability and Math Reminders

Σήμερα

- Catch up
- PseudoRandom Functions (PRFs)
- PseudoRandom Permutations (PRPs)
- Block Ciphers
- Semantic Security

Security in the News

On Friday April 12, Palo Alto disclosed that some versions of PAN-OS are not only vulnerable to remote code execution, but that the vulnerability has been actively exploited to install backdoors on Palo Alto firewalls. A patch is expected to be available on April 14th. The advisory from Palo Alto is here. The CISA advisory is here. Palo Alto has marked this vulnerability as critical and NVD has scored it a 10.0 with CVSSv3. Wallarm currently detects attacks against this vulnerability with no additional configuration required.

What is CVE-2024-3400

A severe command injection vulnerability in the GlobalProtect Gateway feature of PAN-OS versions 10.2, 11.0, and 11.1 underscores the critical importance of API security in devices at the frontline of network connections. The vulnerability, identified as CVE-2024-3400, allows unauthorized users to execute commands as the system administrator, significantly threatening the security of critical infrastructure.

Note: Please ensure that you only use this script for legal and ethical purposes, and only on machines that you have permission to test on.

```
def exploit_firewall(target_ip, payload, root_ca=None):
 url = f"https://{target_ip}/api/"

 data = f"""<?xml version="1.0" encoding="UTF-8"?>
 <request>
 <op cmd="test" />
 <cmd code="ping">{payload}</cmd>
 </request>"""

 headers = {
 "User-Agent": "PAN-OS-Exploit",
 "Content-Type": "application/xml"
 }
```

https://www.volexity.com/blog/2024/04/12/zero-day-exploitation-of-unauthenticated-remote-code-execution-vulnerability-in-globalprotect-cve-2024-3400/

CVE Bonus Challenge

Κάθε CVE που βρείτε αντιστοιχεί σε +1 βαθμό στο μάθημα.
Περιορισμοί:

- Τα CVEs πρέπει να είναι μέσα στο 2024
- Τα CVEs πρέπει να περιέχουν ένα αναγνωριστικό σας (π.χ., name, email, github)

Αν κάνετε claim κάποιο(α) CVE(s) στείλτε μου email με τα αναγνωριστικά

Random Functions and Permutations

Thinking About Mathematical Functions

A function is just a mapping from inputs to outputs:

f_1

x	$f_1(x)$
1	4
2	13
3	12
4	1
5	7

f_2

x	$f_2(x)$
1	1
2	2
3	3
4	4
5	5

f_3

x	$f_3(x)$
1	12
2	3
3	7
4	8
5	10

. . .

~~Which function is *not* random?~~

Thinking About Mathematical Functions

A function is just a mapping from inputs to outputs:

f_1

x	$f_1(x)$
1	4
2	13
3	12
4	1
5	7

f_2

x	$f_2(x)$
1	1
2	2
3	3
4	4
5	5

f_3

x	$f_3(x)$
1	12
2	3
3	7
4	8
5	10

. . .

What is random is the way we *pick* a function

Participation Question

Consider all *functions* of the form F : X -> Y

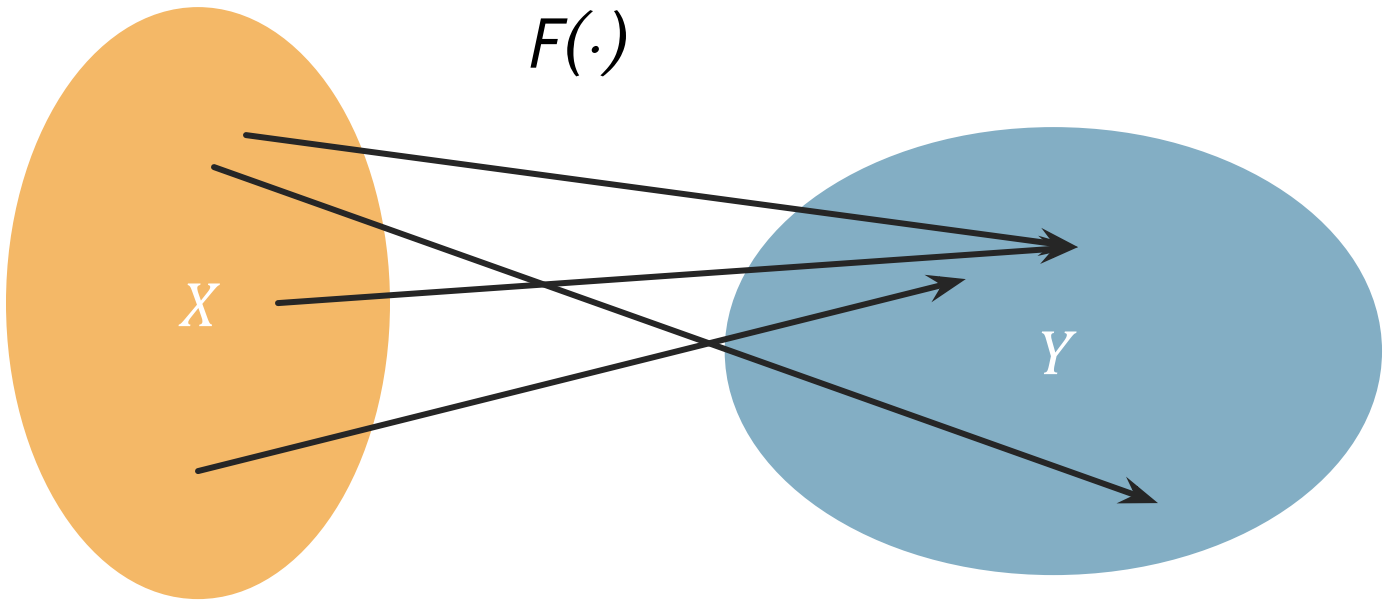
How many possible choices of F are there?

A. |X| * |Y|

B. |X|!

C. $|Y|^{|X|}$

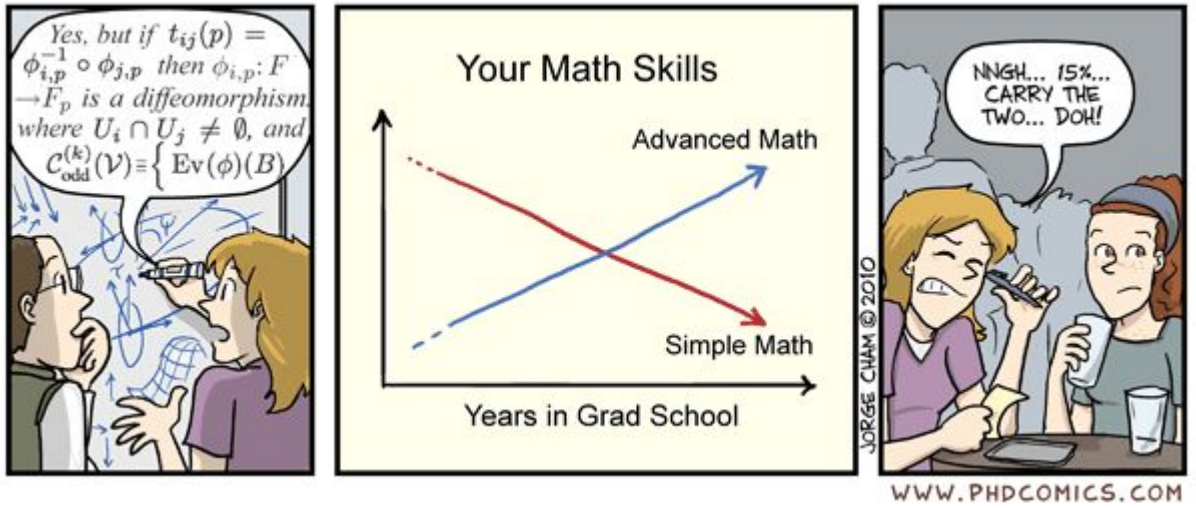
D. $|X|^{|Y|}$



Q: How many functions?

- X = {0, 1, 2} (Domain)
- Y = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} (Range)

10^3 = 1000 possible functions



Encryption with Functions

- Alice chooses f: $\{0,1\}^b \rightarrow \{0,1\}^b$ at random from *all possible functions* from $\{0,1\}^b$ to $\{0,1\}^b$
- Alice gives Bob the inverse, f^{-1}
- Given message $m \in \{0,1\}^b$:
  - Alice sends f(m) to Bob
  - Bob decrypts using f^{-1}

Participation Question

Is this a correct cipher?

A. Yes

B. No

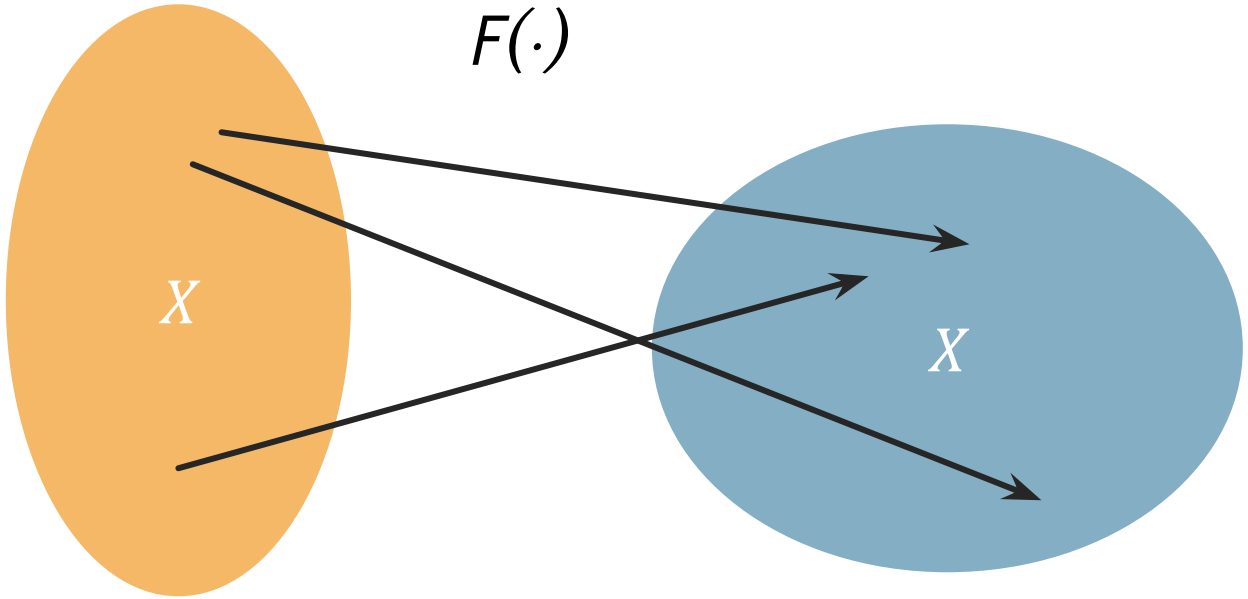
C. I'm not sure

Correctness

$$\forall m \in M, k \in K : D(k, E(k, m)) = m$$

Permutations: Definition

- f: X -> X
- A permutation:
  - is a function -> maps **every element** of its domain to **one** element of its range
  - Every element in the range is **mapped** to by exactly one element of the domain
- In math terms: f is one-to-one
  - $\forall x_1, x_2 . f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$
- Colloquially, f is a shuffling of X



Participation Question

Consider all *permutations* of the form F : X -> X

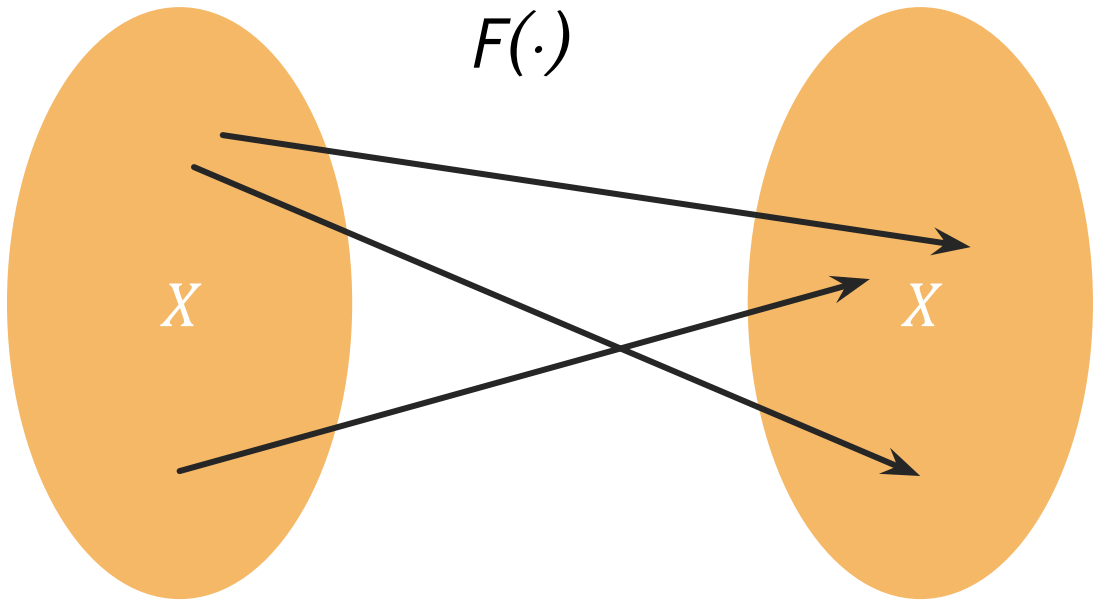
How many possible choices of F are there?

A. $2|X|$

B. $|X|^2$

C. $|X|! \approx (\frac{|X|}{e})^{|X|}$

D. $|X|^{|X|}$



Better Encryption Scheme?

- Alice chooses f: $\{0,1\}^b$ -> $\{0,1\}^b$ at random from all possible ***permutations*** from $\{0,1\}^b$ to $\{0,1\}^b$
- Alice gives Bob the inverse, f^{-1}
- Given message m ∈ $\{0,1\}^b$:
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<u>Participation Question</u>

Is this a correct cipher?

A. Yes

B. No

C. I'm not sure

Good cipher?

Better Encryption Scheme?

- Alice chooses f: $\{0,1\}^b$ -> $\{0,1\}^b$ at random from all possible ***permutations*** from $\{0,1\}^b$ to $\{0,1\}^b$
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Did we bypass “bad news” theorem?

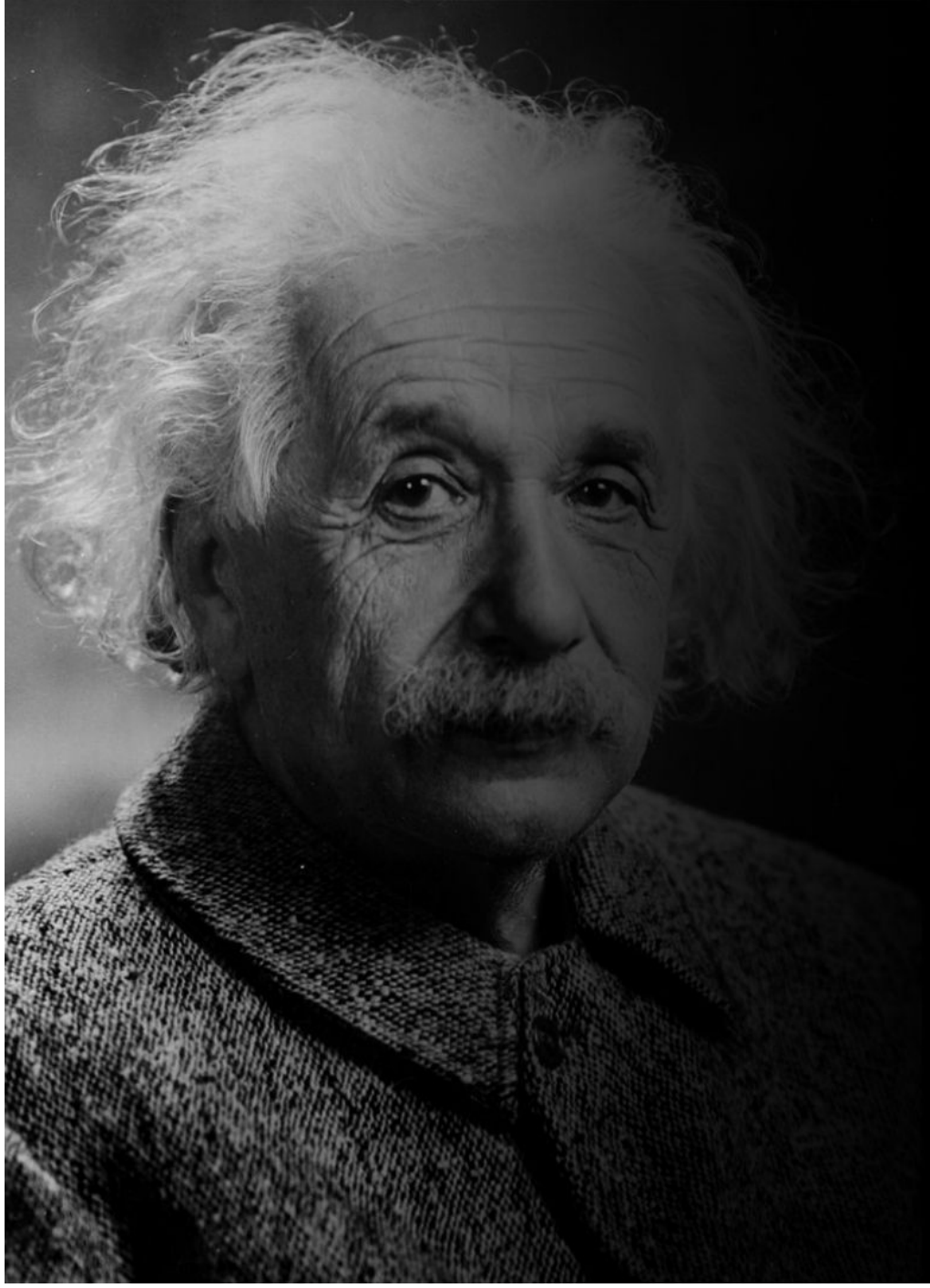
Computational security

The system can be *practically* (not perfectly) indecipherable

- Security is only preserved against efficient adversaries running in polynomial time and space, with access to randomness
- Adversaries can succeed with a very small probability (small enough that it is essentially impossible)
  - Ex: Probability of guessing a large randomly chosen value

"A scheme is secure if every Probabilistic Polynomial Time (PPT) adversary succeeds in breaking the scheme with only negligible probability"

PseudoRandom Functions and Permutations



God does not play dice with the universe.

ALBERT EINSTEIN

Pseudorandomness (overloaded term)

A **pseudorandom** sequence of numbers is one that appears to be statistically random, despite having been produced by a completely deterministic and repeatable process.[1] Simply put, the problem is that many of the sources of randomness available to humans (such as rolling dice) rely on physical processes not readily available to computer programs.

In theoretical computer science, a distribution is **pseudorandom** against a class of adversaries if no adversary from the class can distinguish it from the uniform distribution with significant advantage.[6] This notion of pseudorandomness is studied in computational complexity theory and has applications to cryptography.

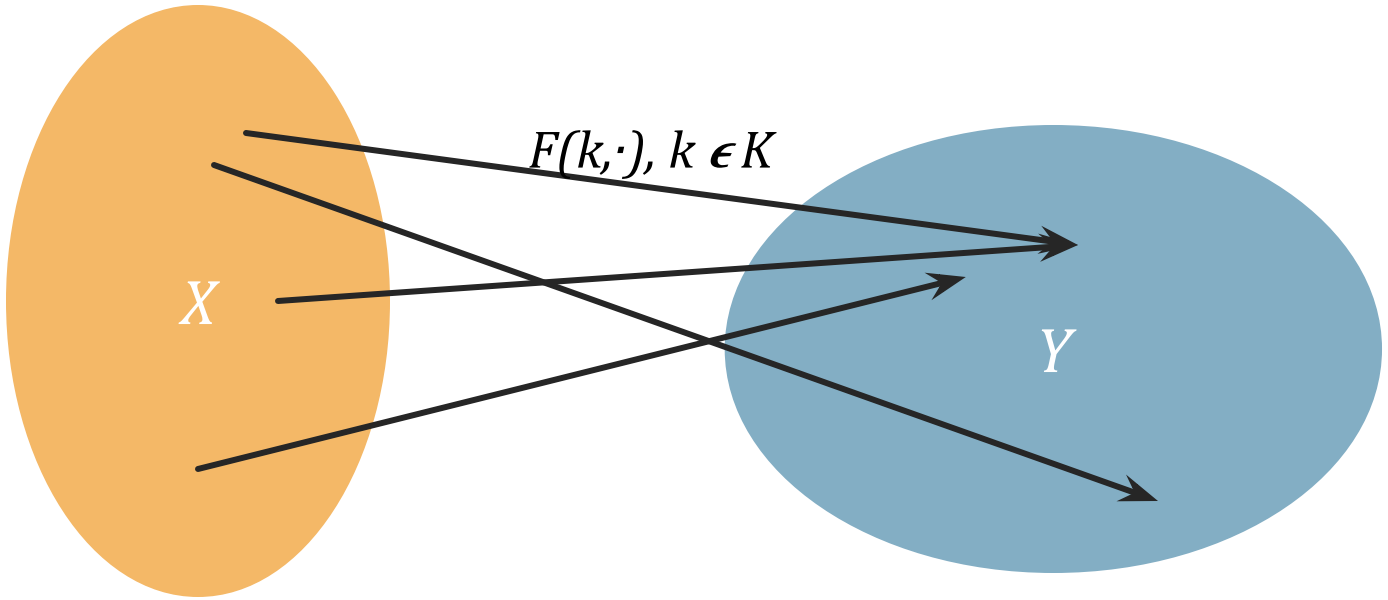
https://en.wikipedia.org/wiki/Pseudorandomness

PRFs

Pseudo Random Function (PRF) defined over (*K*, *X*, *Y*):

$$F : K \times X \rightarrow Y$$

such that there exists an "efficient" algorithm to evaluate *F(k,x)*



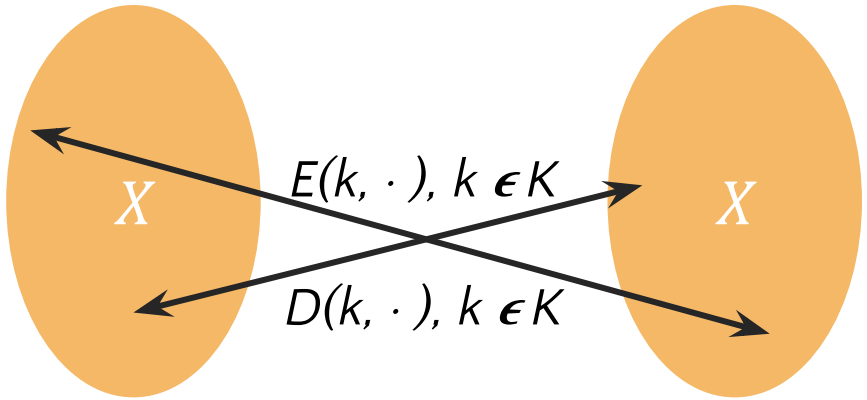
PRPs

Pseudo Random Permutation (PRP) defined over (K,X)

$$E : K \times X \rightarrow X$$

such that:

1. Exists "efficient" deterministic algorithm to evaluate *E(k,x)*
2. The function *E(k, ·)* is one-to-one
3. Exists "efficient" inversion algorithm *D(k,y)*



Let's Use Today's State-of-the-Art PRP (AES)

Question: what if we want to encrypt more than 128 bits?

PRFs and PRPs Are Still Math Functions!

- They map inputs to outputs
  - Conceptually, just a giant table
- They are not stateful!
- They are not randomized!

What if someone manages to invert our PRF/PRP function?

One Way Functions

In computer science, a **one-way function** is a function that is easy to compute on every input, but hard to invert given the image of a random input.

A function $f : \{0, 1\}^* \to \{0, 1\}^*$ is **one-way** if f can be computed by a polynomial-time algorithm, but any polynomial-time randomized algorithm F that attempts to compute a pseudo-inverse for f succeeds with negligible probability.

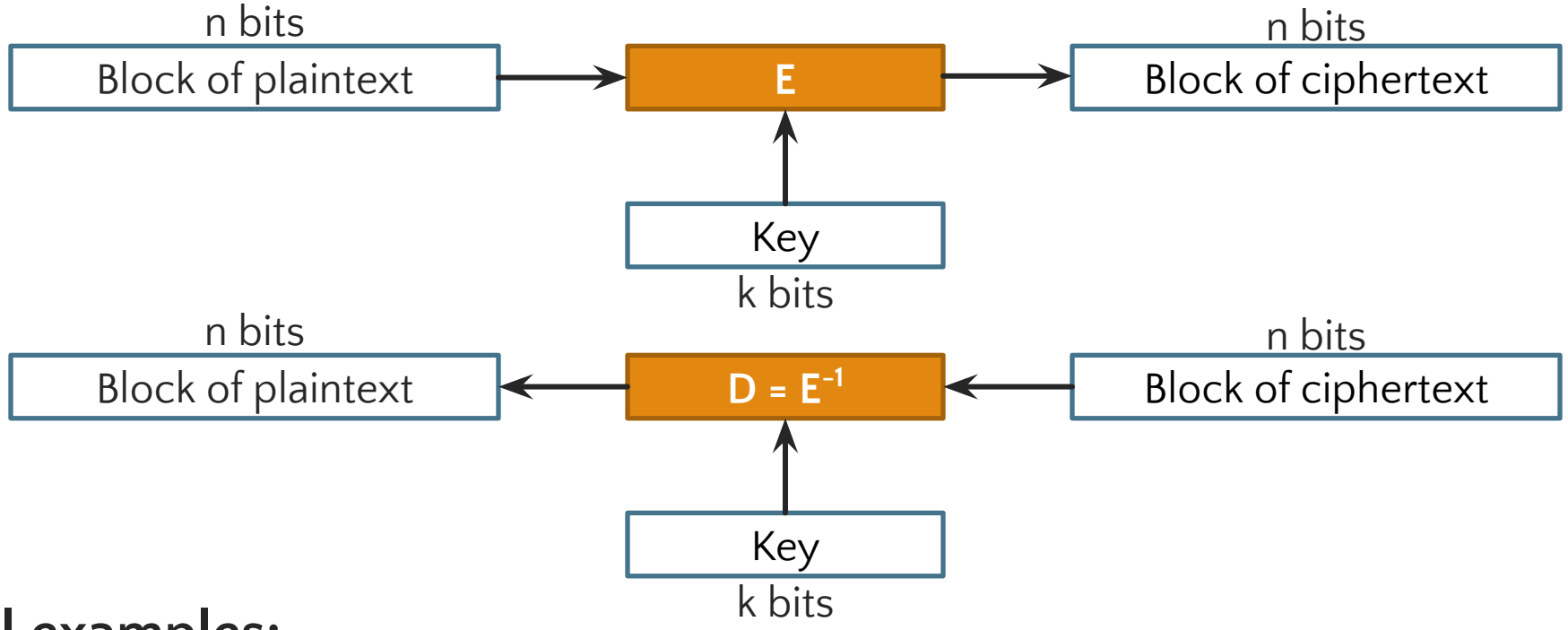
The existence of such one-way functions is still an open conjecture. Their existence would prove that the complexity classes P and NP are not equal, thus resolving the foremost unsolved question of theoretical computer science.[1]

https://en.wikipedia.org/wiki/One-way_function

Block Ciphers (aka practical PRPs)

Block Cipher ≈ PRP

Block ciphers are the crypto work horse



Canonical examples:

1. ~~DES: n = 64 bits k = 56 bits~~
2. ~~3DES: n = 64 bits k = 168 bits~~
3. AES: n = 128 bits k = 128, 192, 256 bits

History of DES

- **1970s**: Horst Feistel designs Lucifer at IBM

  key = 128 bits, block = 128 bits

- **1973:** NBS asks for block cipher proposals.

  IBM submits variant of Lucifer.

- **1976:** NBS adopts DES as federal standard

  key = 56 bits, block = 64 bits

- **1997:** DES broken by exhaustive search

DES Challenge

Message	`The unkn`	`own mess`	`age is:`	`xxxxxxxx`
Ciphertext	c_1	c_2	c_3	c_4

Goal: find $k \in \{0,1\}^{56}$ s.t. $DES(k, m_i) = c_i$ for i=1,2,3

How expensive is it to reveal $DES^{-1}(k, c_4)$?

1976	DES adopted as federal standard		
1997	Distributed search	3 months	
1998	EFF deep crack	3 days	$250,000
1999	Distributed search	22 hours	
2006	COPACOBANA (120 FPGAs)	7 days	$10,000

⇒ 56-bit keys should not be used (128-bit key ⇒ 2^{72} days)

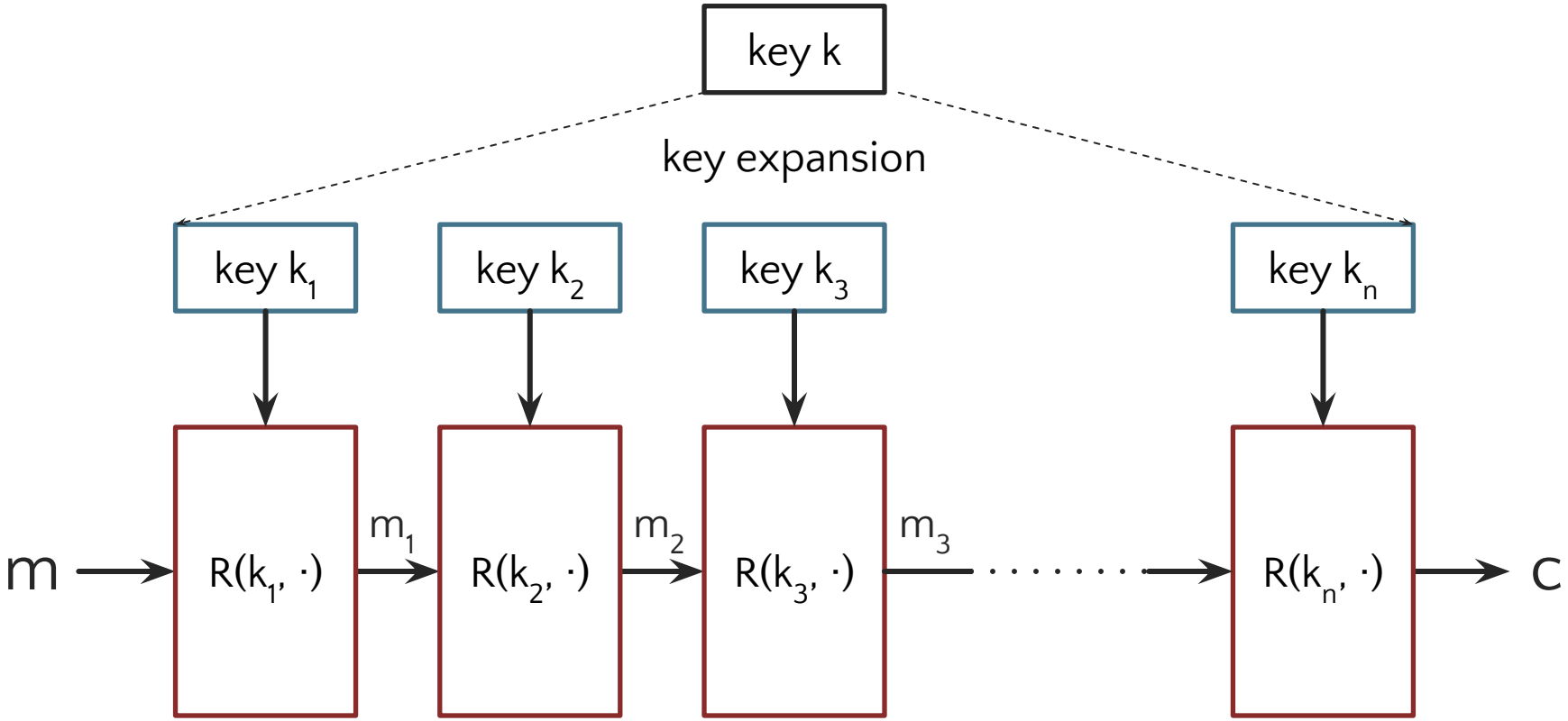
Advanced Encryption Standard (AES): The Process

- **1997**: DES broken by exhaustive search
- **1997**: NIST publishes request for proposal
- **1998**: 15 submissions
- **1999**: NIST chooses 5 finalists
- **2000**: NIST chooses Rijndael as AES
  (developed by Daemen and Rijmen at K.U. Leuven, Belgium)

Key sizes: 128, 192, 256 bits

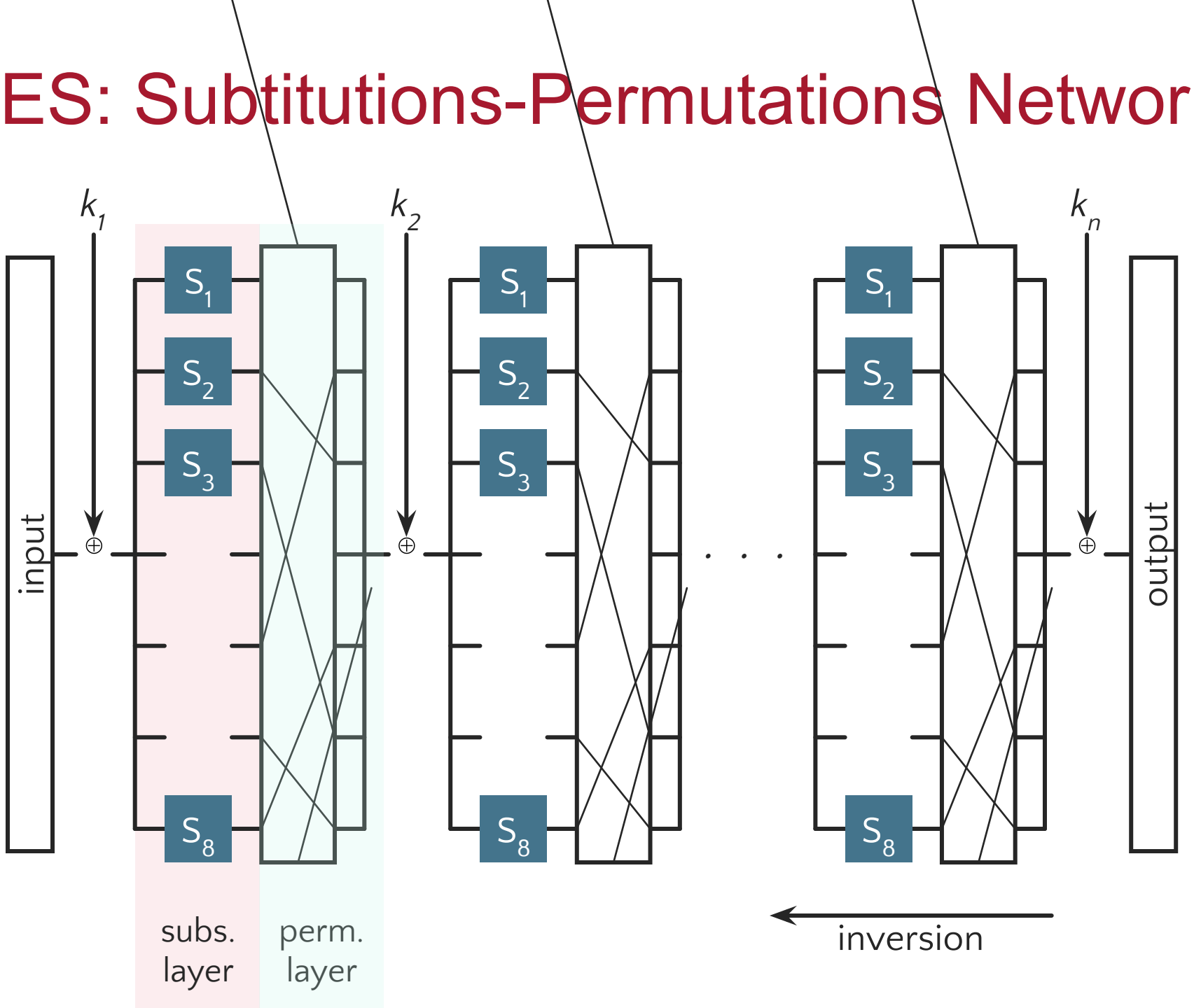
Block size: 128 bits

Block Ciphers Built by Iteration



R(k, m) is called a *round function*
invoked up to n times
Ex: DES (n=16), 3DES (n=48), AES128 (n=10)

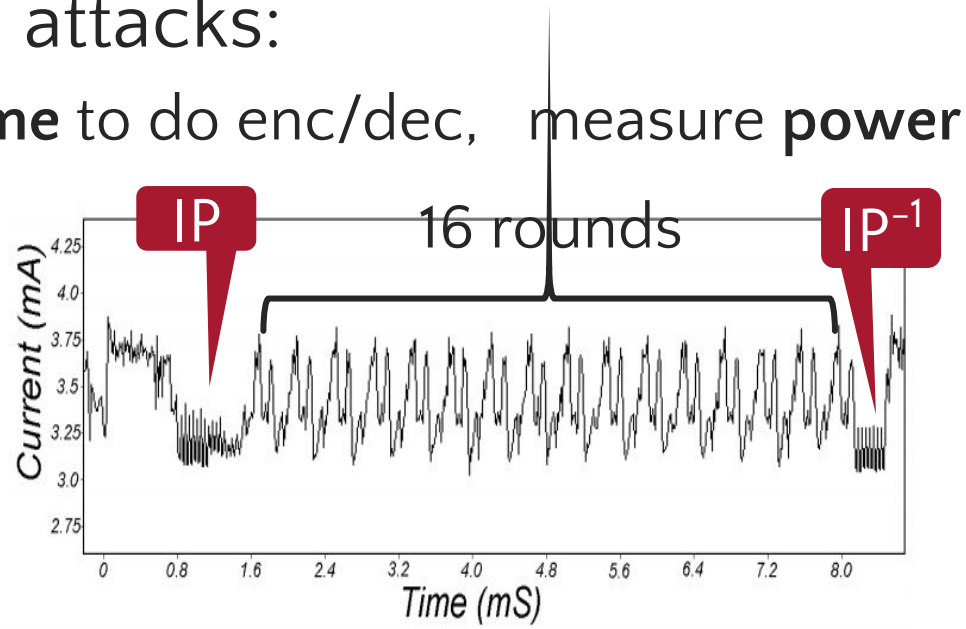
AES: Subtitutions-Permutations Network



Attacks on the Implementation

1. Side channel attacks:
   - Measure **time** to do enc/dec, measure **power** for enc/dec

smartcard



[Kocher, Jaffe, Jun, 1998]

Card is doing DES

2. Fault attacks:
   - Computing errors in the last round expose the secret key k

⇒ never implement crypto primitives yourself …

Can We Encrypt Using Block Ciphers?

- Is a block cipher a secure encryption algorithm?
- Are they useful?


Semantic Security

Goldwasser and Micali, Turing Award 2012

What Is a Secure Encryption Alg.?

Attacker's abilities: obtains one ciphertext (for now)

Attempt #1: Attacker cannot recover key

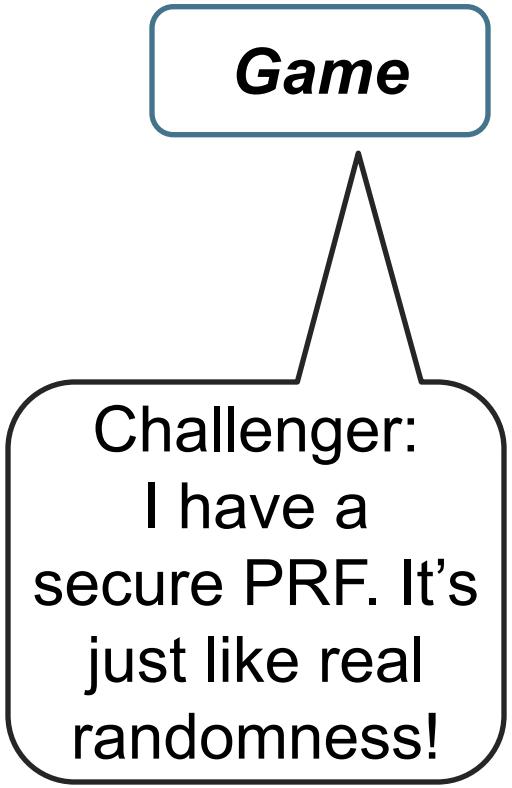
Insufficient: Consider $E(k,m) = m$

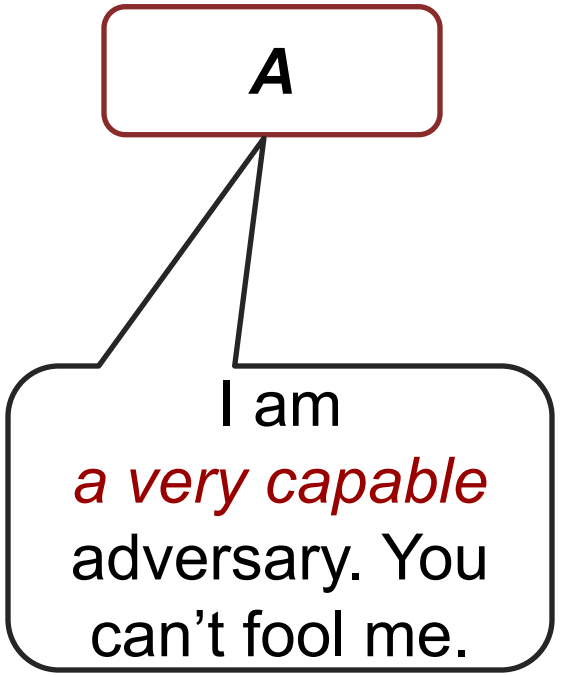
Attempt #2: Attacker cannot recover all of plaintext

Insufficient: Consider $E(k,m_0 \,||\, m_1) = m_0 \,||\, F(k,m_1)$

Recall Shannon's Intuition:
c (output of E) should reveal no information about *m*

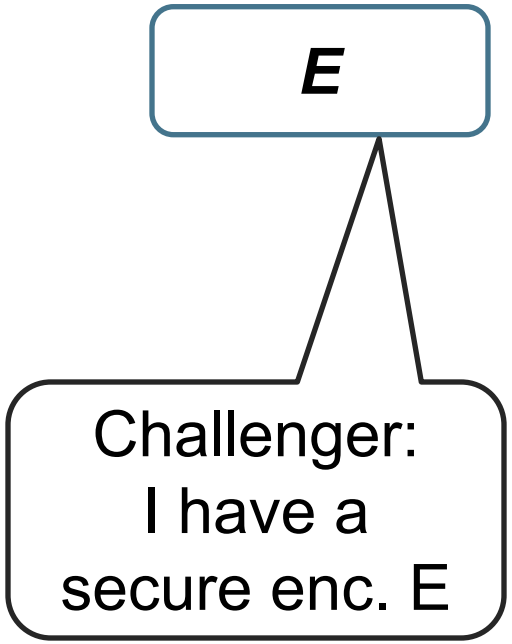
Defining Security: Adversarial Indistinguishability Game

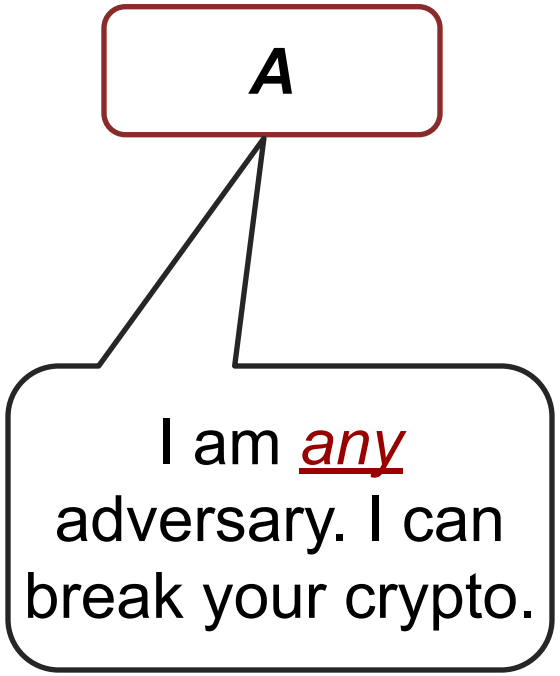




Security Games

Adversarial Indistinguishability Game





Semantic Security Intuition

1. ***A*** sends m_0, m_1 s.t. $|m_0|=|m_1|$ to the challenger

m_0, m_1

2. Challenger computes:
   $c = E(k, m_b)$

   where b is a coin flip.

   Sends back c.

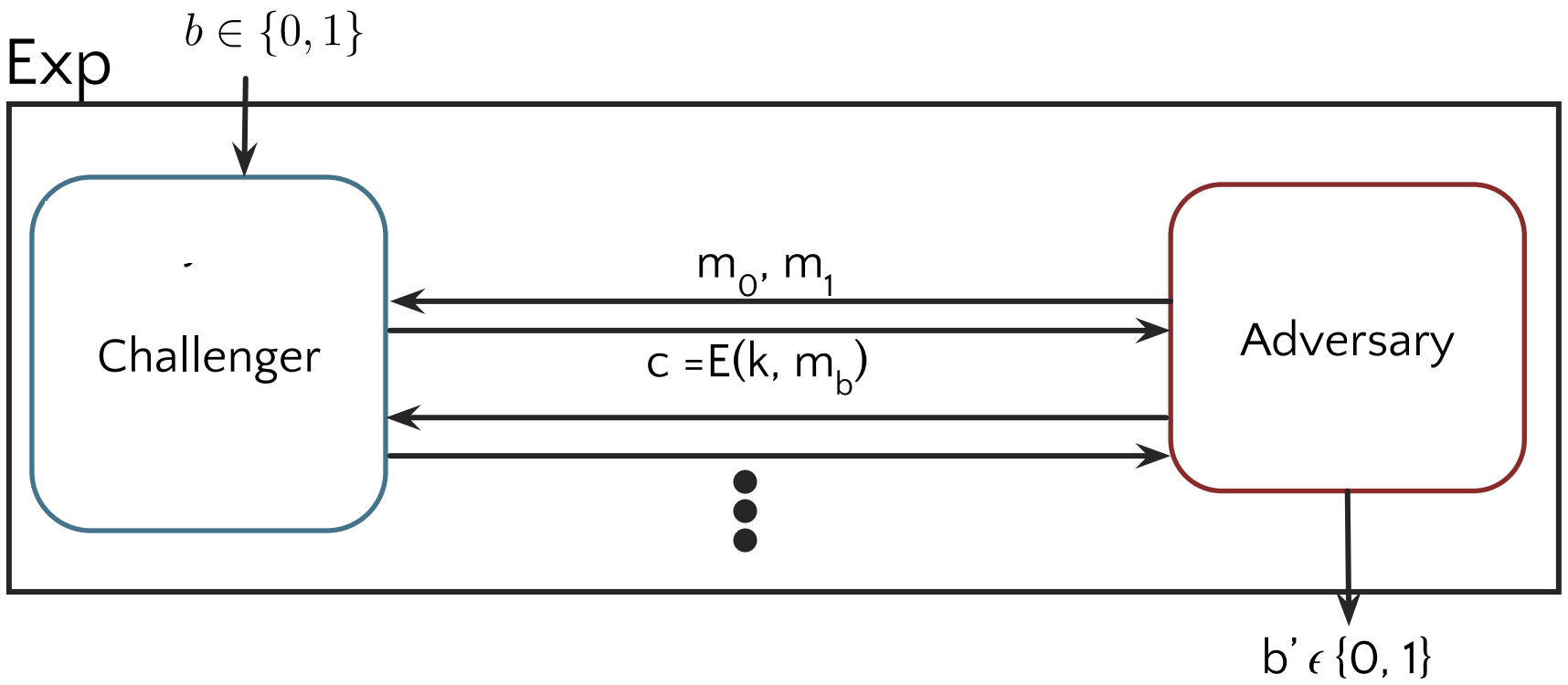
c

3. ***A*** guesses which message was encrypted

4. Challenger wins if ***A*** does no better than guessing

Semantic Security *IND-CPA* Game

For *b = 0,1* define experiment *Exp(b)* as:



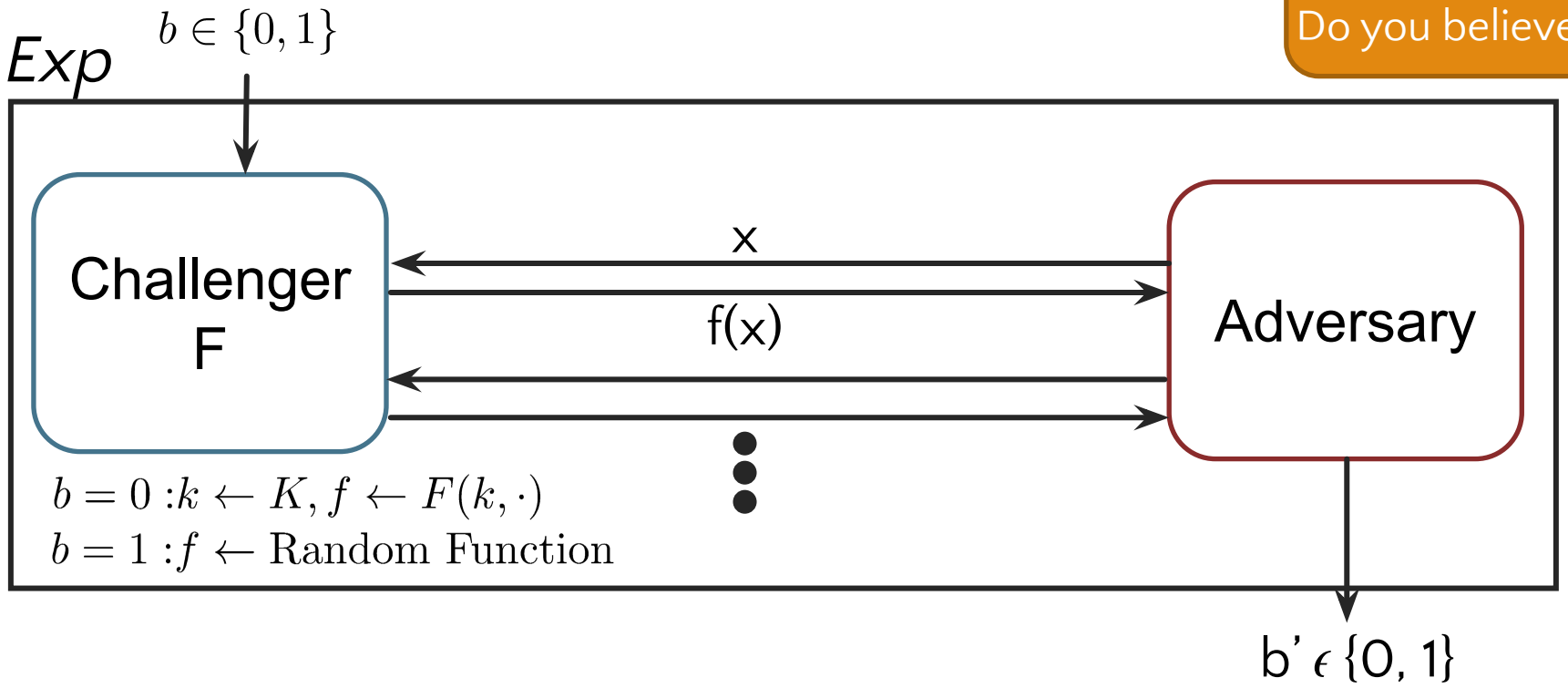
Defn: E is IND-CPA secure if for all efficient A:

$Adv_{IND-CPA}[A, E] := Pr[Exp(1) = 1] - Pr[Exp(0) = 1] < \varepsilon$

PRF Security Game

For *b = 0,1* define experiment *Exp(b)* as:

Important
This is a definition!
Do you believe it captures PRF?

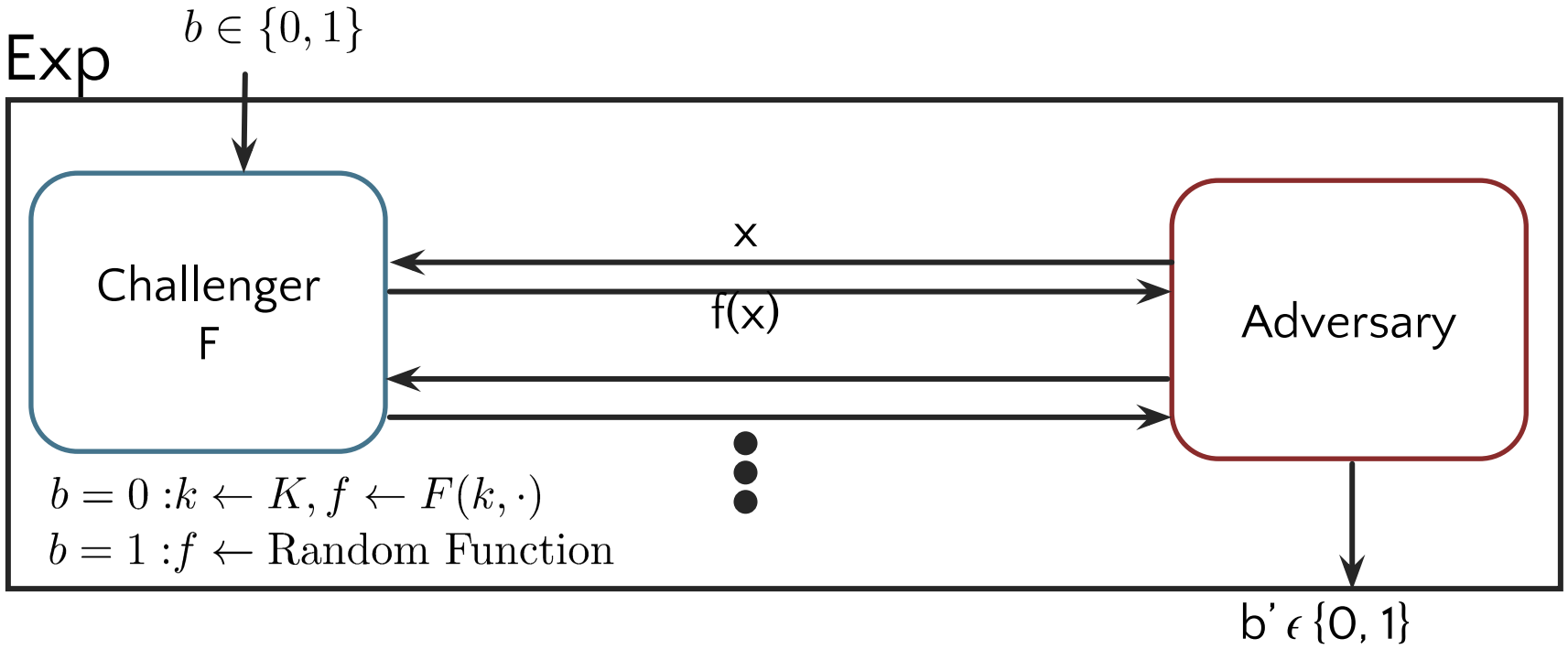


Defn: F is a secure PRF if for all efficient *A*:

$Adv_{PRF}[A, F, q] := |Pr[Exp(0) = 1] - Pr[Exp(1) = 1]| < \varepsilon$

where A makes at most q queries

Sanity Check: Guessing

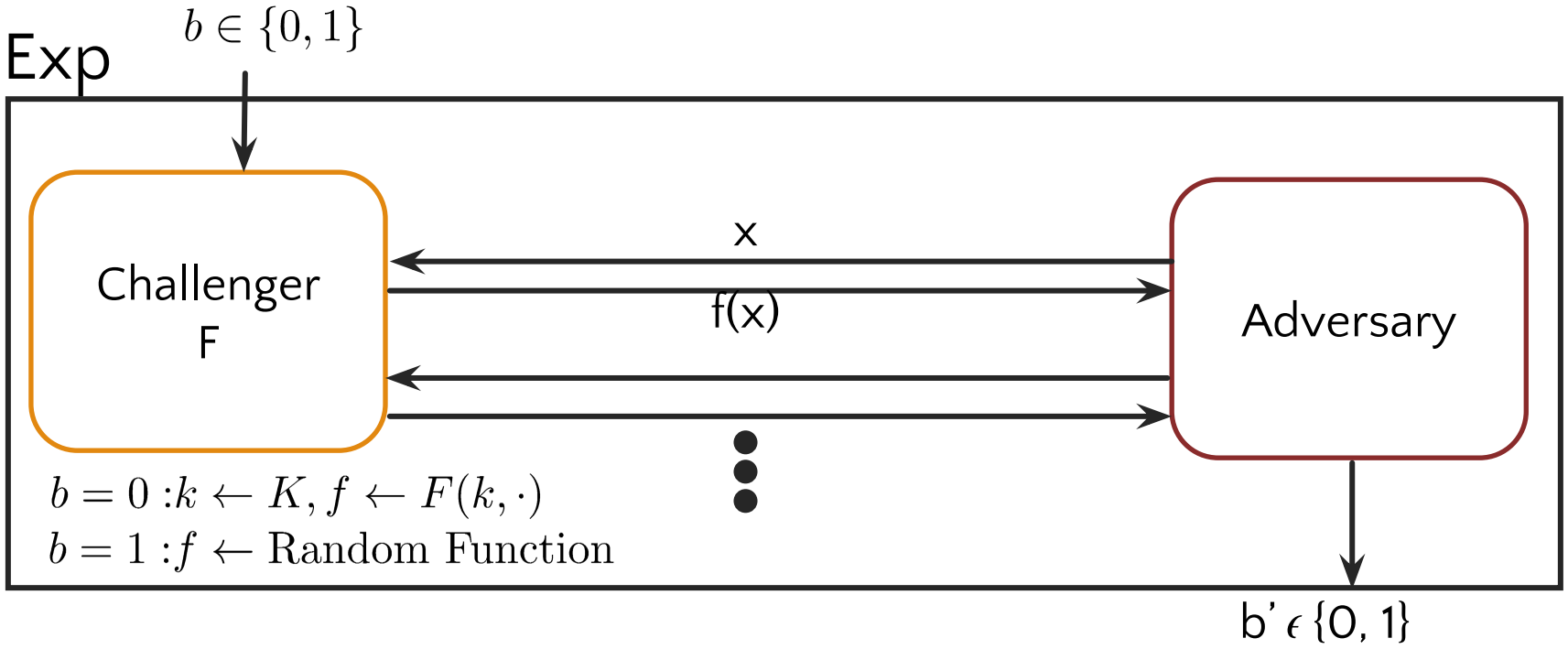


$$\mathbf{Adv}_{PRF}[A,F] := |\Pr[Exp(0)=1] - \Pr[Exp(1)=1]| < \epsilon$$

Suppose the adversary simply flips a coin. Then
Pr[Exp(0) = 1] = 0.5 Pr[Exp(1) = 1] = 0.5

Then: $\text{Adv}_{\text{PRF}}[\boldsymbol{A},\boldsymbol{F}] = |.5 - .5| = 0$

Example: Non-Negligible

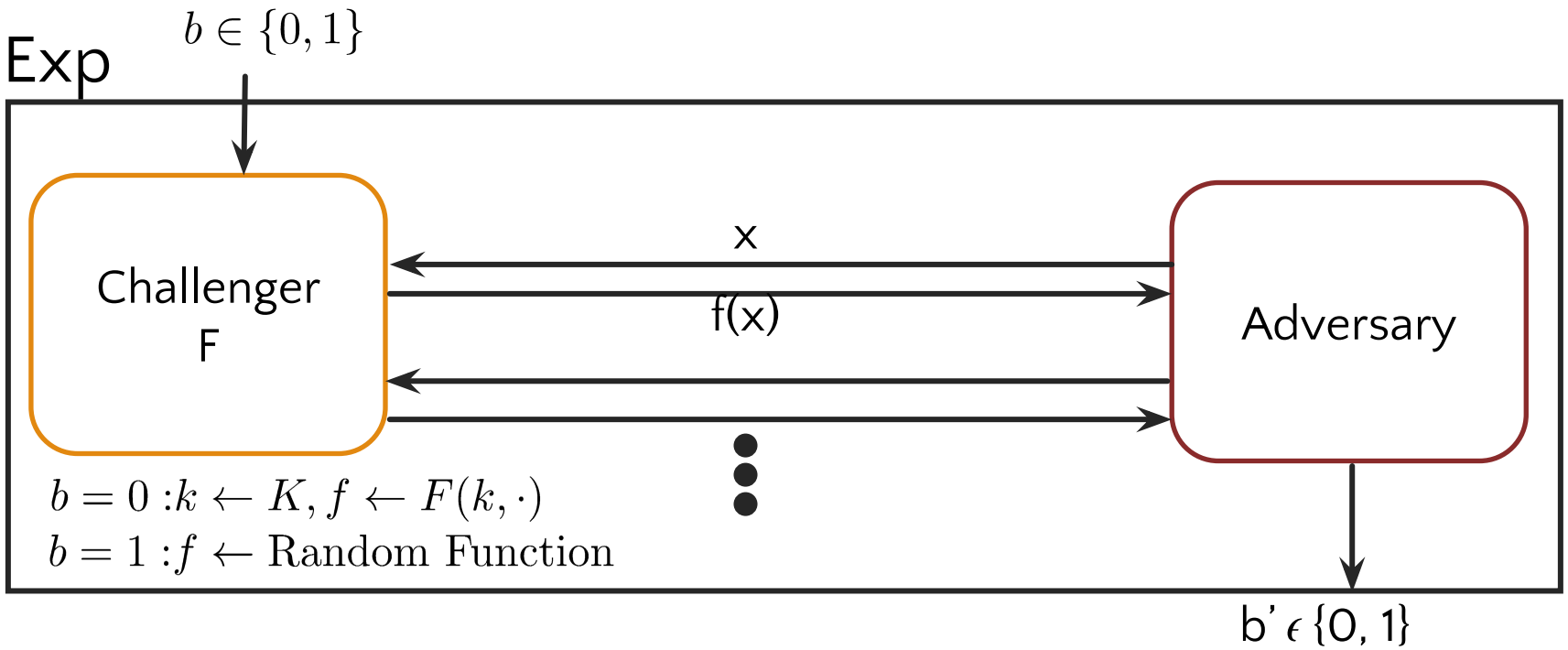


$$\mathbf{Adv}_{PRF}[A,F] := |\Pr[Exp(0)=1] - \Pr[Exp(1)=1]| < \epsilon$$

Suppose the PRF is slightly broken, say:
Pr[Exp(0) = 1] = 0.2 Pr[Exp(1) = 1] = 0.8

Then: $\text{Adv}_{\text{PRF}}[\boldsymbol{A},\boldsymbol{F}] = |0.2 - 0.8| = 0.6$

Example: Wrong More Than 50%



$$\mathbf{Adv}_{PRF}[A,F] := |\Pr[Exp(0)=1] - \Pr[Exp(1)=1]| < \epsilon$$

Suppose the adversary is almost always wrong, say:
Pr[Exp(0) = 1] = 0.8 Pr[Exp(1) = 1] = 0.2

Then: $Adv_{PRF}[\boldsymbol{A},\boldsymbol{F}] = |0.8 - 0.2| = 0.6$

Guessing wrong > 50% of the time yields an alg. to guess right

Participation Question

Let $F : K \times X \rightarrow \{0,1\}^{128}$ be a secure PRF.

Is the following G a secure PRF?

$$G(k,x) = \begin{cases} 0^{128} & \text{if } x = 0 \\ F(k,x) & \text{otherwise} \end{cases}$$

A. No, it is easy to distinguish G from a random function

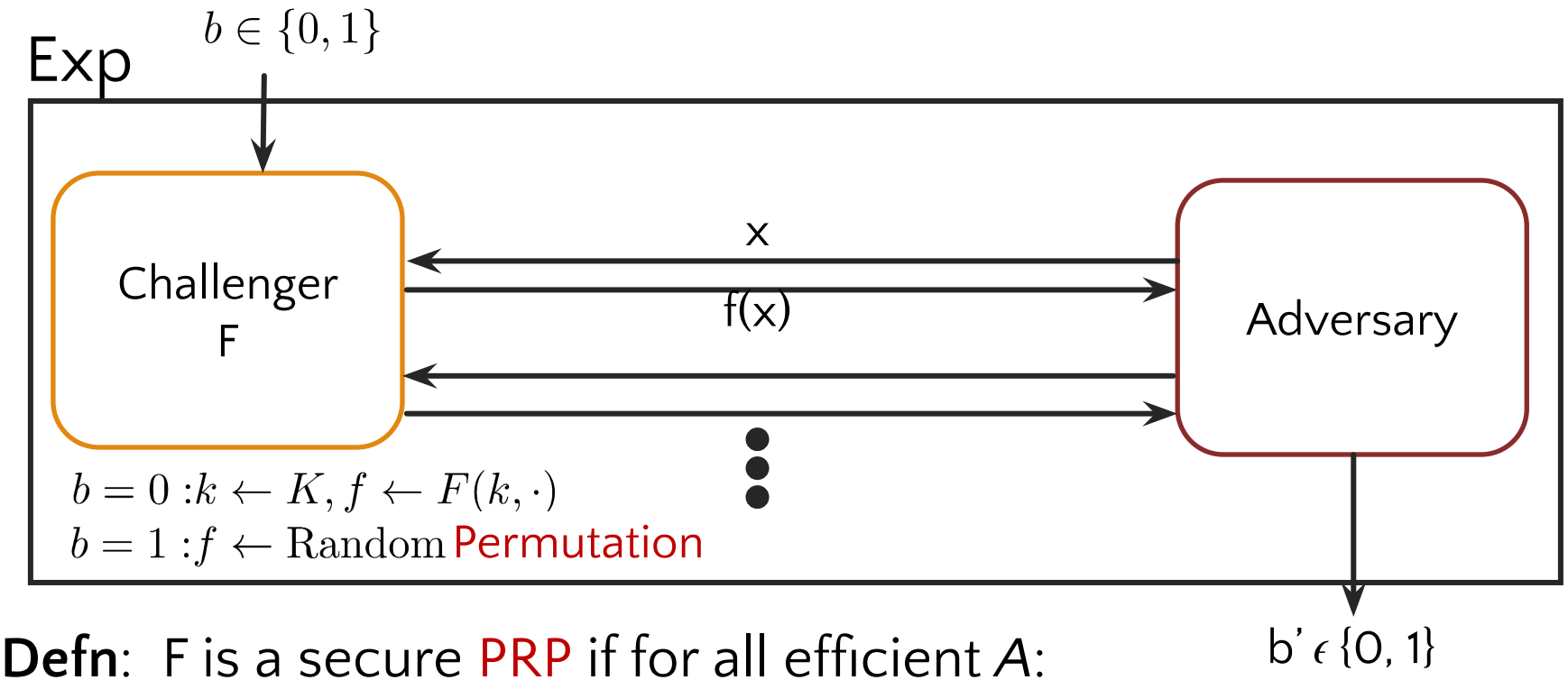
B. No, G might map more than one input to 0^{128}

C. Yes, an attack on G would also break F

D. It depends on F

PRP Security Game

For *b = 0,1* define experiment *Exp(b)* as:



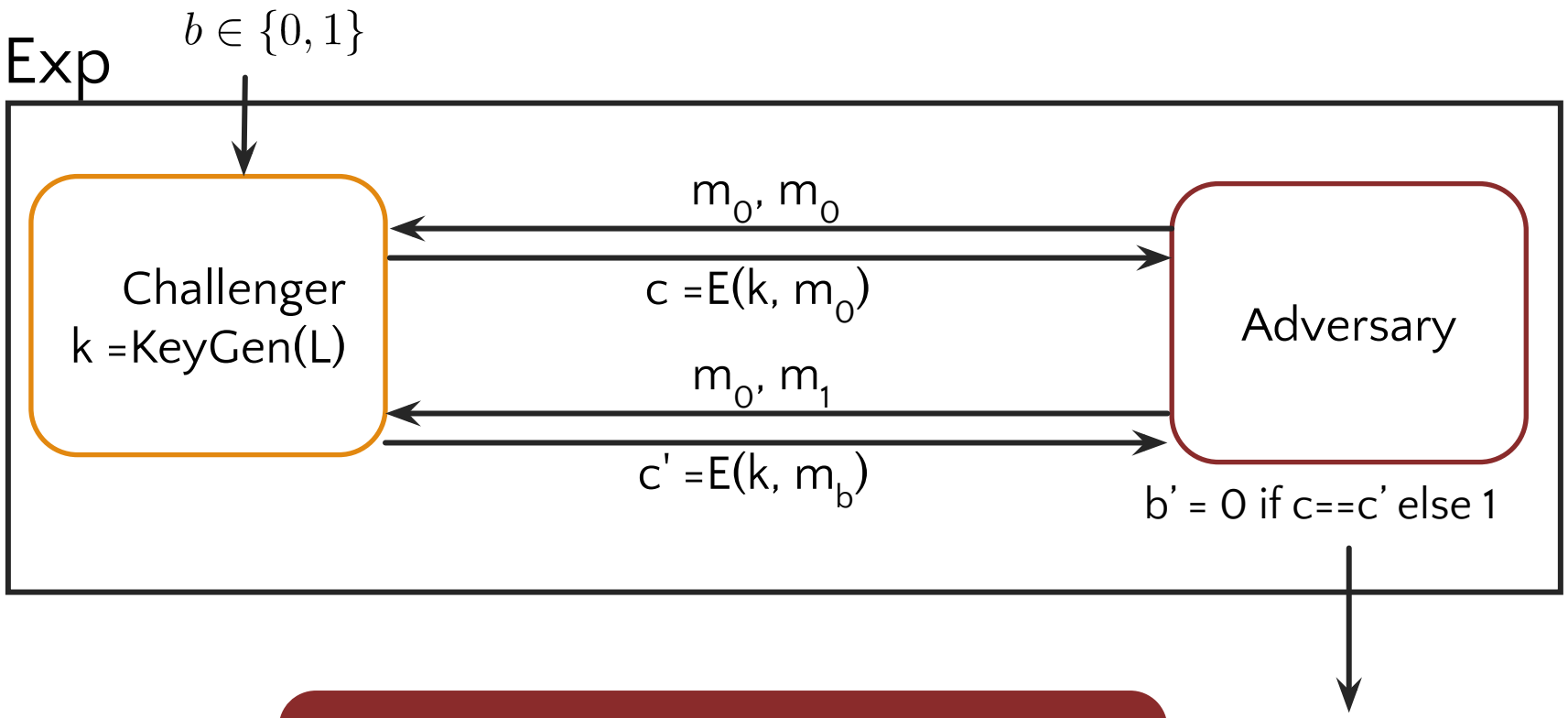
Defn: F is a secure PRP if for all efficient *A*:

$$\mathbf{Adv}_{\mathrm{PRP}}[A,F] := |\Pr[Exp(0)=1] - \Pr[Exp(1)=1]| < \epsilon$$

Let’s Apply This Definition of Security

Breaking Deterministic, Stateless Encryption

For *b = 0,1* define experiment *Exp(b)* as:



Encryption must be randomized or be stateful!

How do we encrypt more data safely??

Next time!

Ευχαριστώ και καλή μέρα εύχομαι!

Keep hacking!