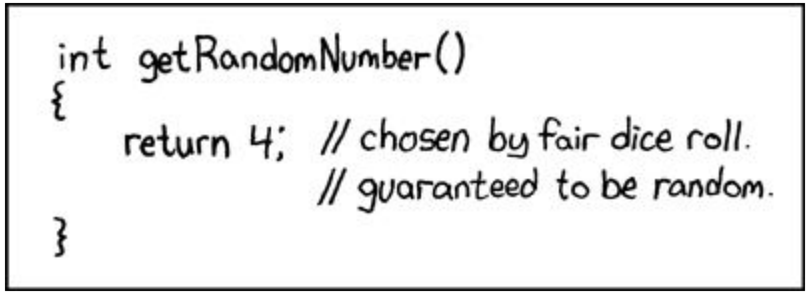
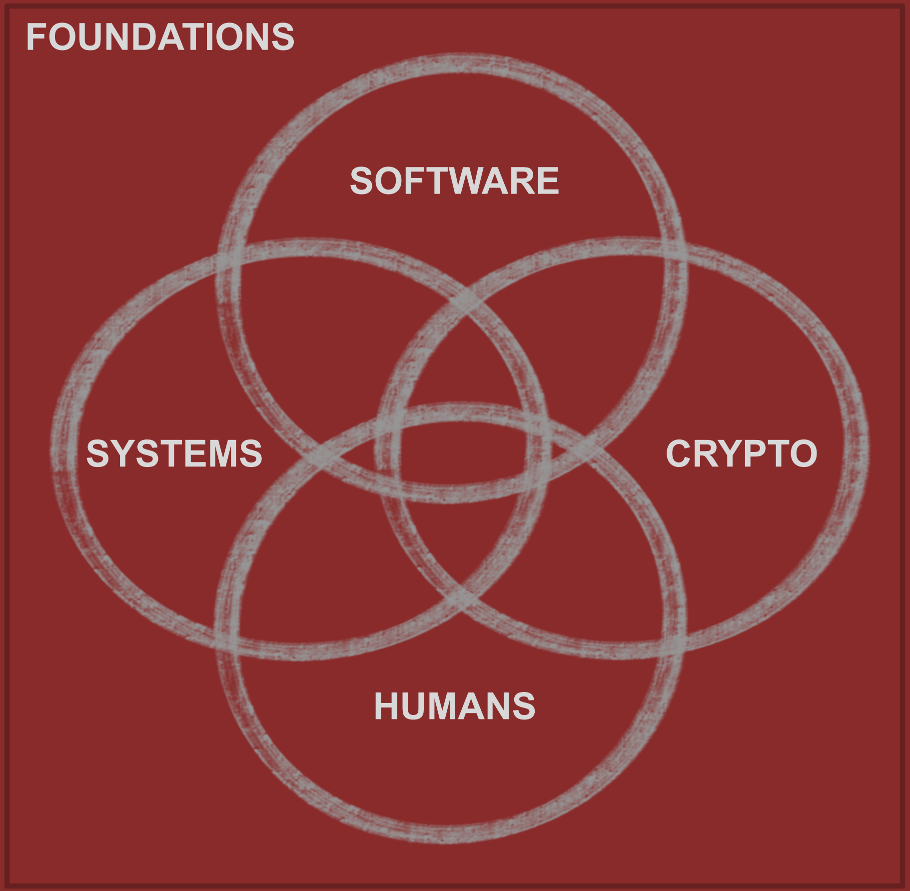
Διάλεξη #12 - On Randomness

Εθνικό και Καποδιστριακό Πανεπιστήμιο Αθηνών

Εισαγωγή στην Ασφάλεια

Θανάσης Αυγερινός





Huge thank you to [David Brumley] from Carnegie Mellon University for the guidance and content input while developing this class (some slides from Dan Boneh @ Stanford!)

Ανακοινώσεις / Διευκρινίσεις

Ερωτήσεις:

- Cryptographically Secure vs Obscure - ποια η διαφορά;
- Είσαι σίγουρος ότι είναι OK να επιτρέπουμε το κλειδί "0" σε OTP; Ή αντίστοιχα κλειδιά τα οποία έχουν συγκεκριμένα patterns;

Την προηγούμενη φορά

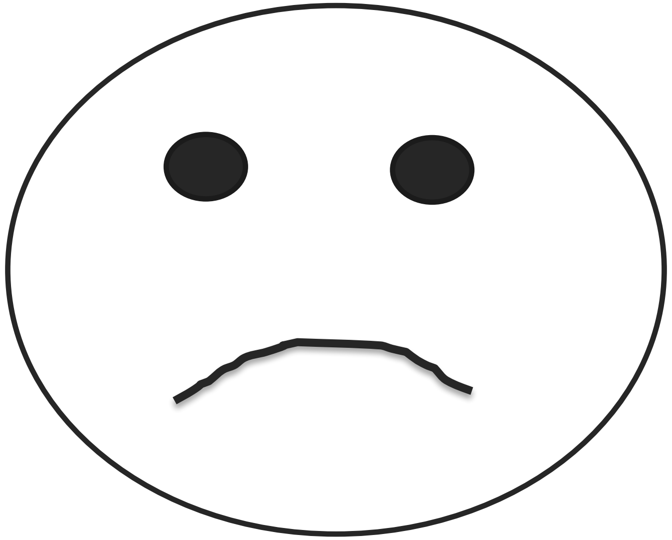
- About cryptography
- Terminology
- Traditional ciphers
- One-time pad

Σήμερα

- Problems with just OTP
- Randomness and Pseudorandomness
- Probability and Math Reminders
- PseudoRandom Functions (PRFs)
- PseudoRandom Permutations (PRPs)

The “Bad News” Theorem

Theorem: Perfect secrecy requires |K| >= |M|



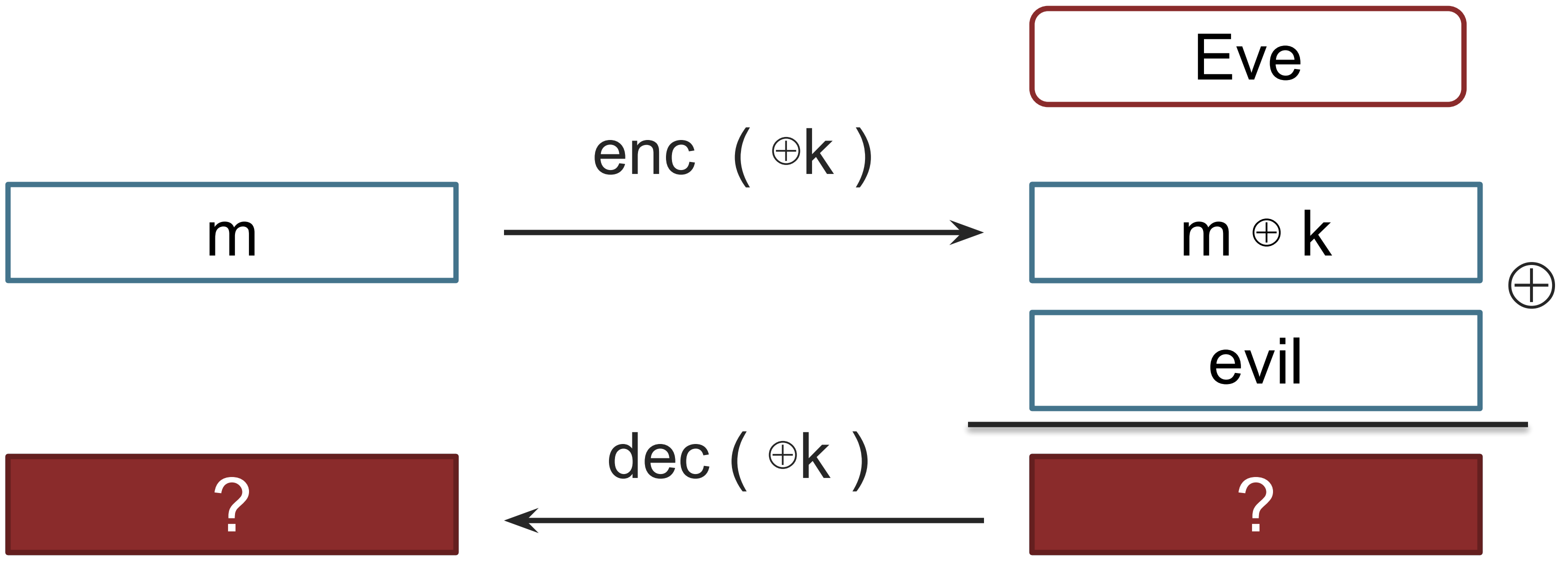
In practice, we usually shoot for *computational security*

More bad news

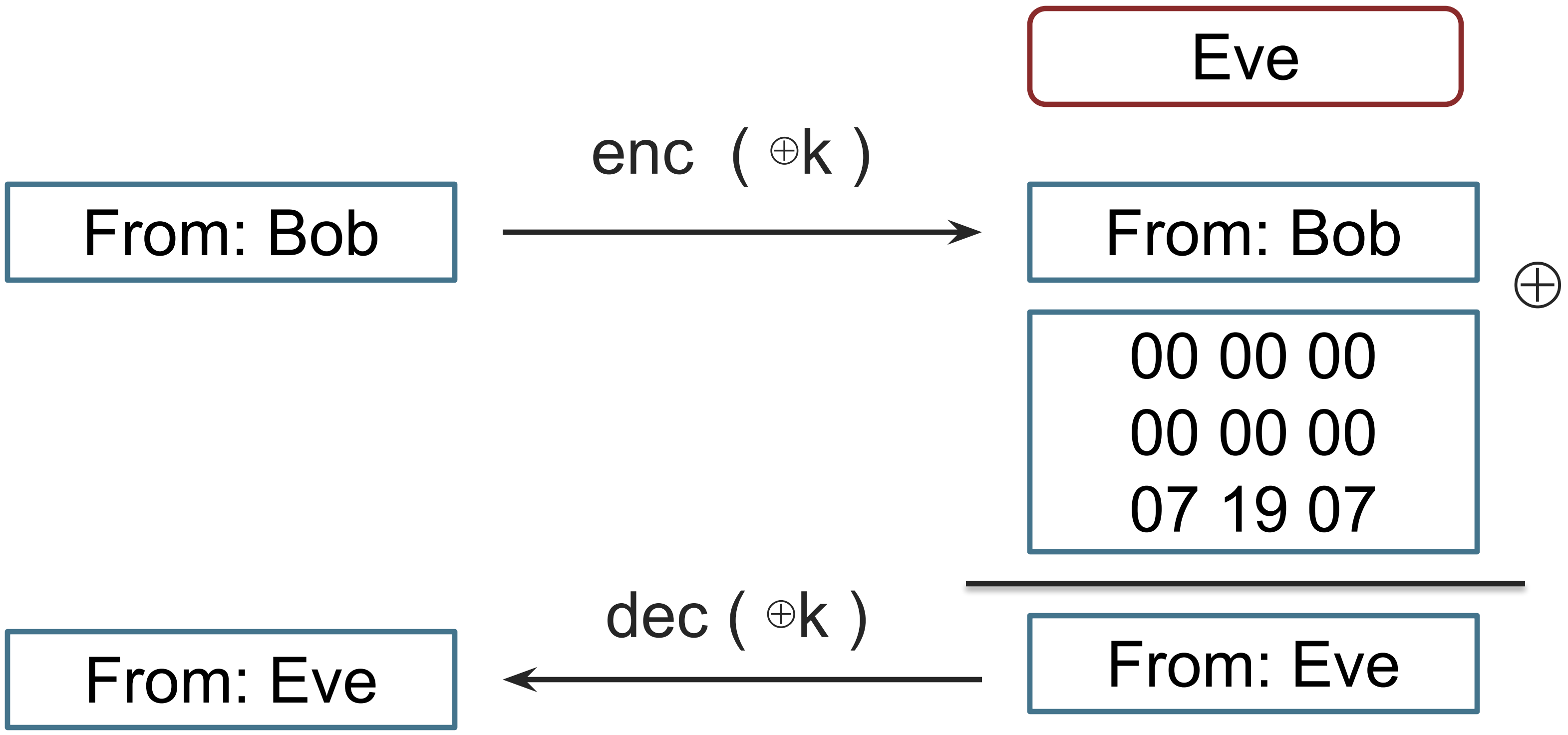
The OTP provides perfect secrecy …

… but is that enough?

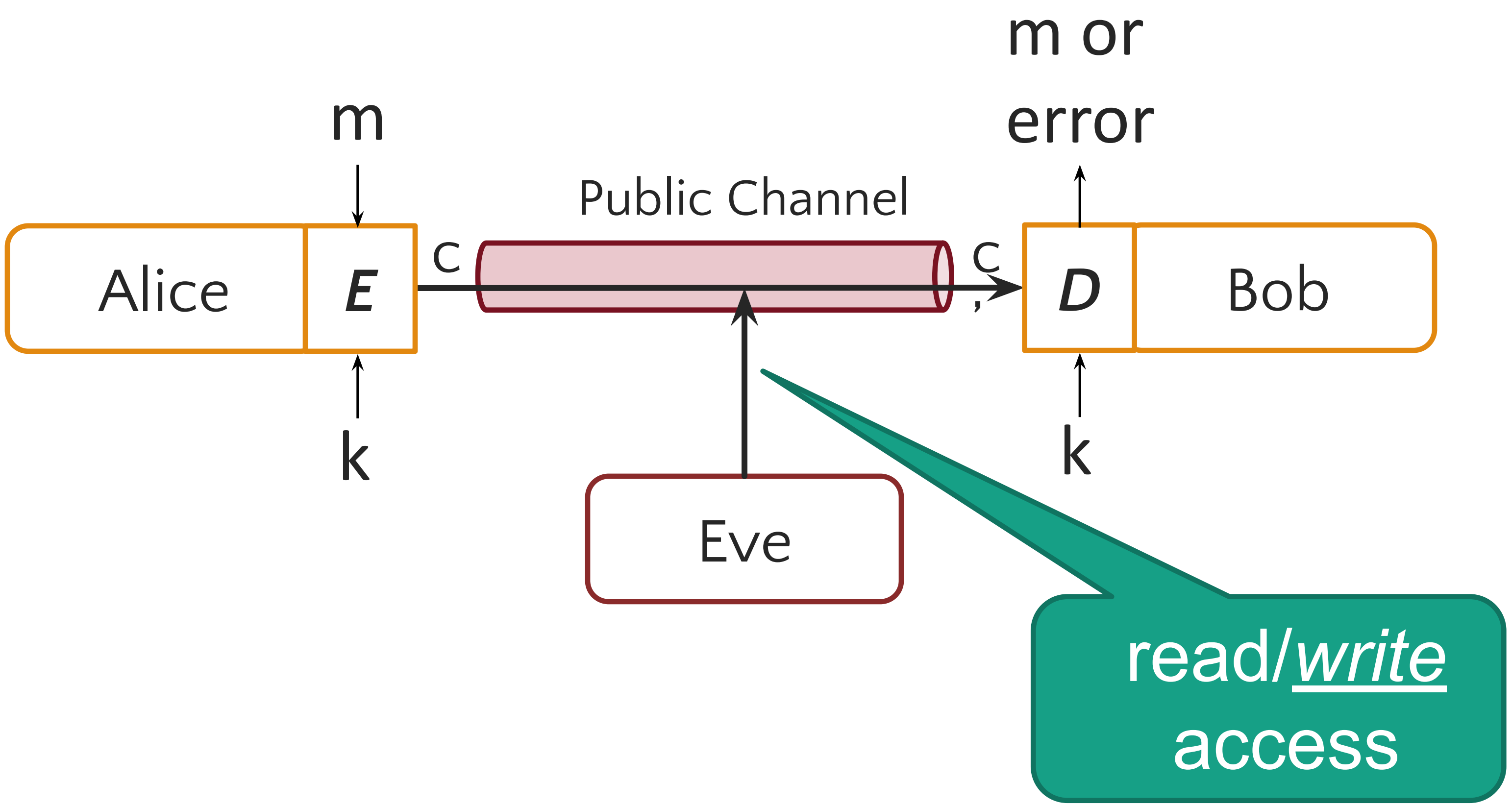
No Integrity



No Integrity



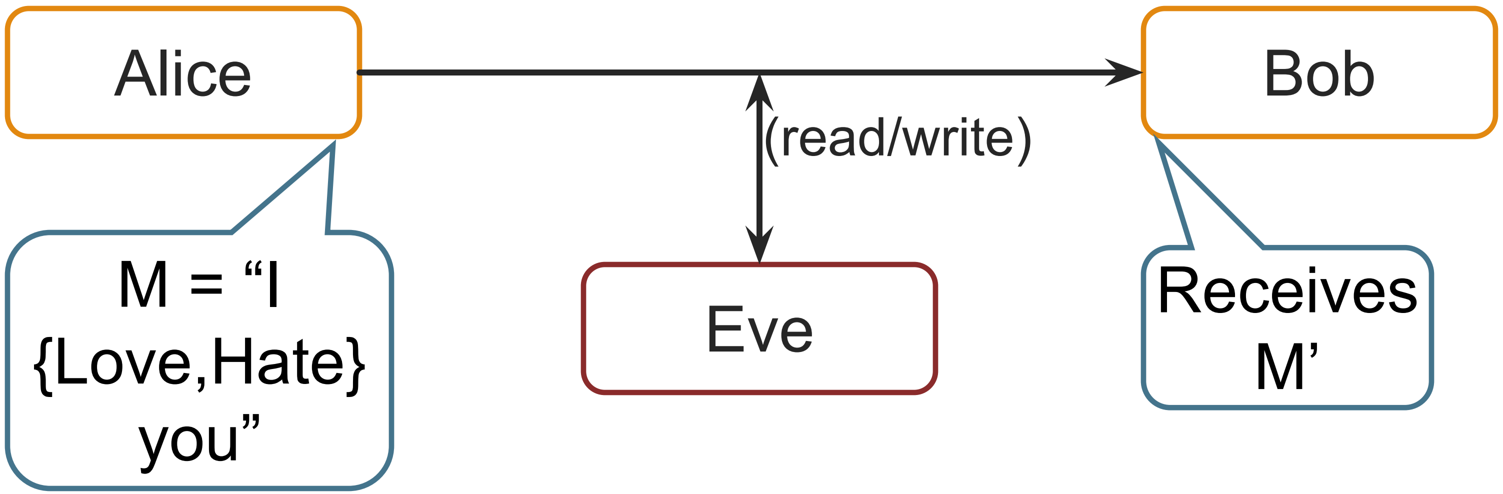
Our Goal: Secure Communication



Sub Goal 2: ***Integrity***

Eve should not be able to alter *m* without detection

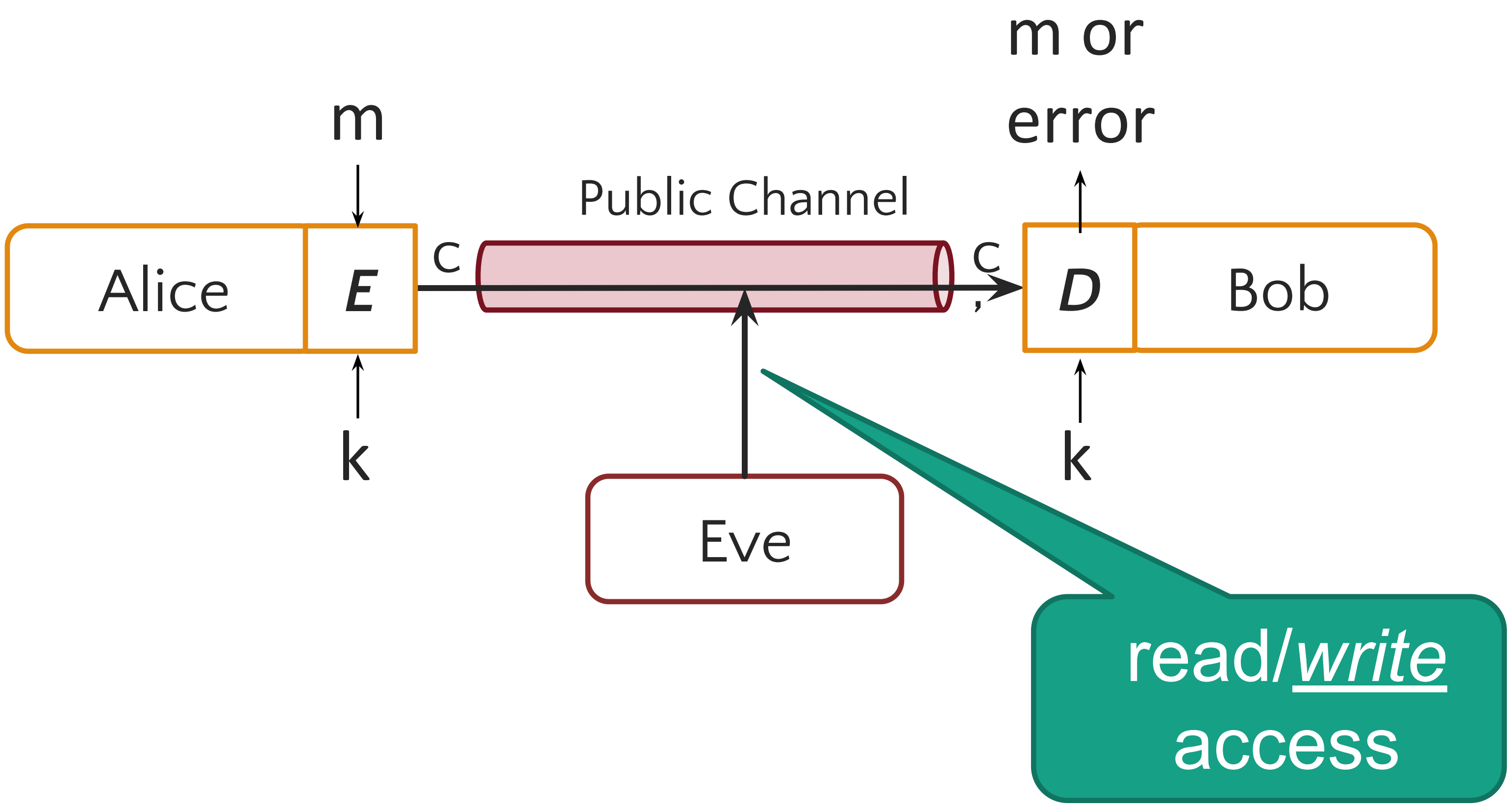
Detecting Modifications



Bob should be able to determine if M’=M

Ex: Eve should not be able to change Alice’s message without detection (even if Eve doesn’t know content of M)

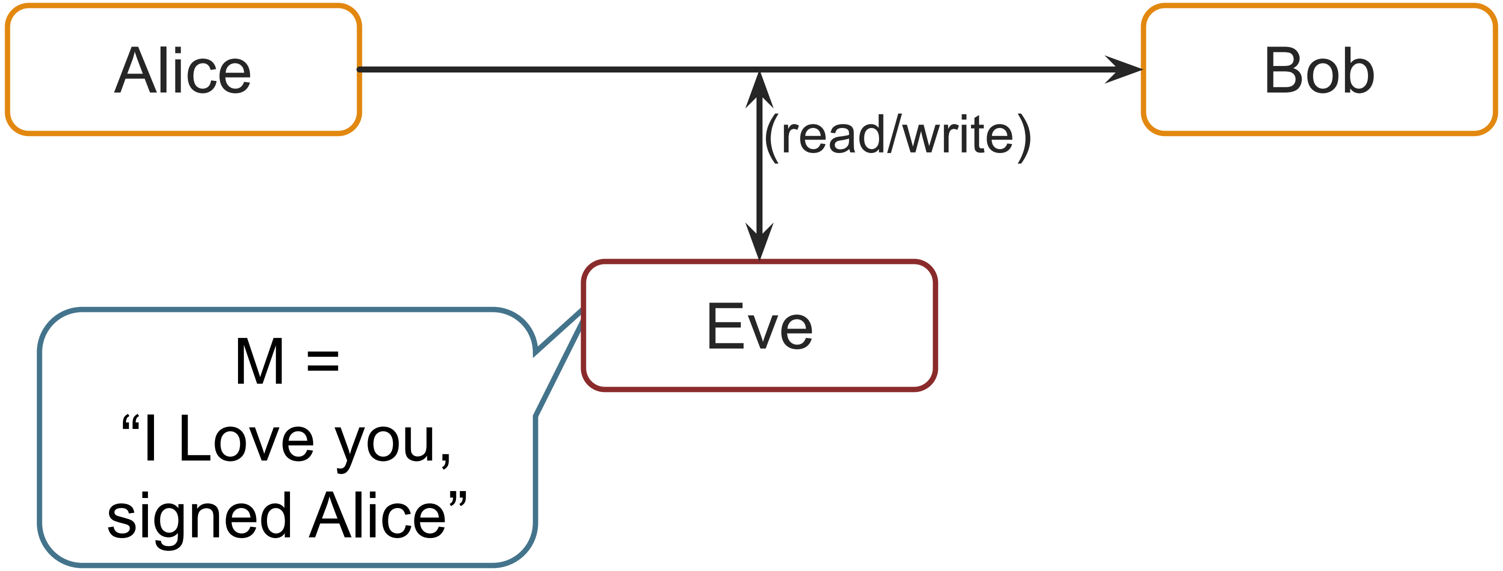
Our Goal: Secure Communication



Sub Goal 3: ***Authenticity***

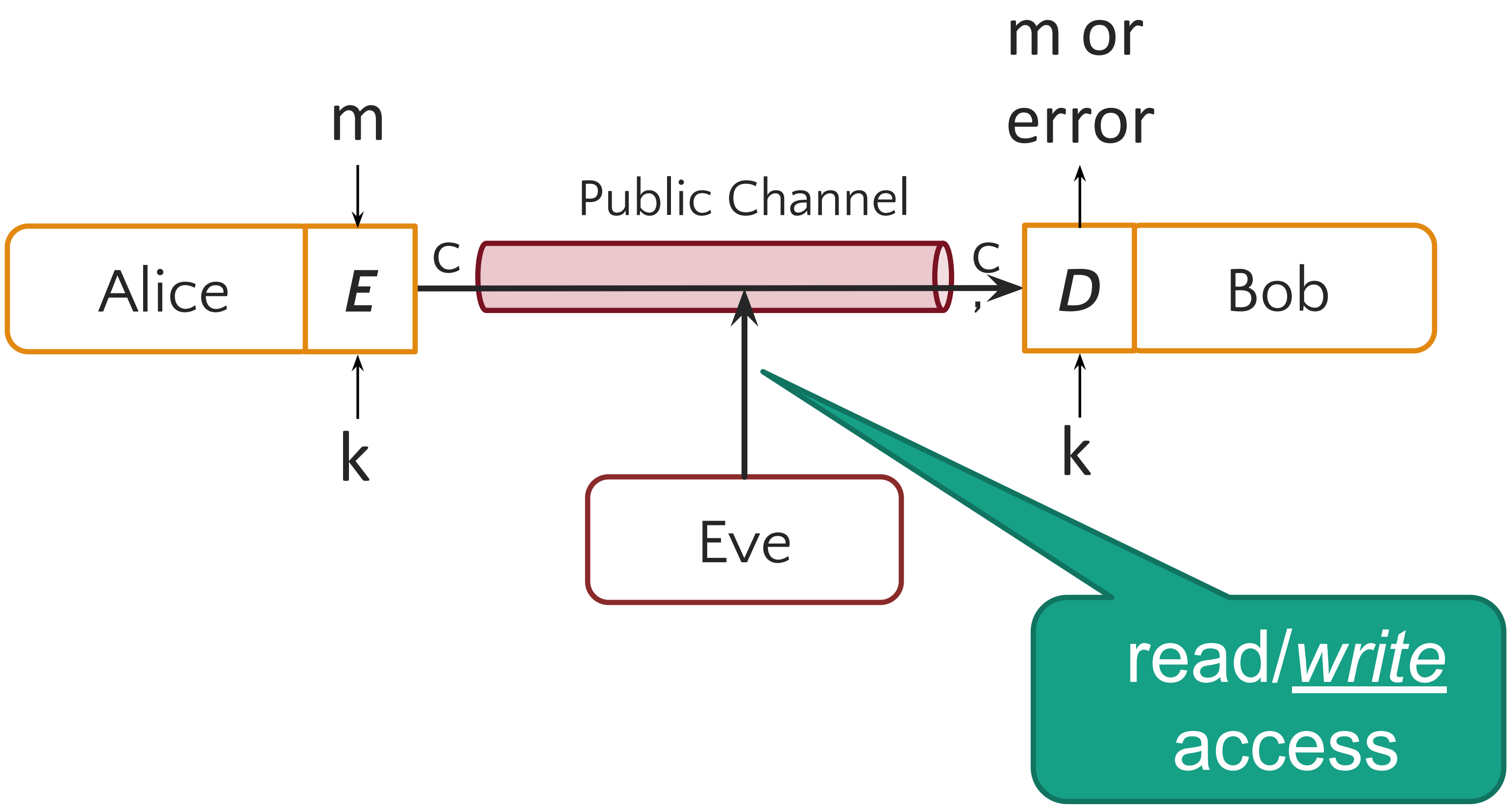
Eve should not be able to forge messages as Alice

Detecting Message Injection



Bob should be able to determine whether M was sent by Alice

Our Goal: Secure Communication



Secure Communication means:
Secrecy, Integrity, and Authenticity

Still open: the pieces we need for secure communication

Everyone shares *same* secret k

Only 1 party has a secret

	Symmetric Trust Model	Asymmetric Trust Model
Message Privacy	Private key encryption • Stream Ciphers • Block Ciphers	Asymmetric encryption (aka public-key encryption)
Message Authenticity and Integrity	Message Authentication Code (MAC)	Digital Signature Scheme

Principle 1: All algorithms are *public* (Kerckhoffs's Principle)
Principle 2: Security is determined *only* by key size
Principle 3: If you roll your own, it will be *insecure*

A Crucial Ingredient: Randomness!

Crucial Ingredient: Randomness

- Explicit usage
  - Generate secret keys
  - Generate random “nonces” for encryption (more later on)
- Less obvious usage:
  - Generate passwords for new users
  - Shuffle cards in a poker game or votes in an election
  - Choose which work items to audit for correctness

Insecure Randomness: C rand()

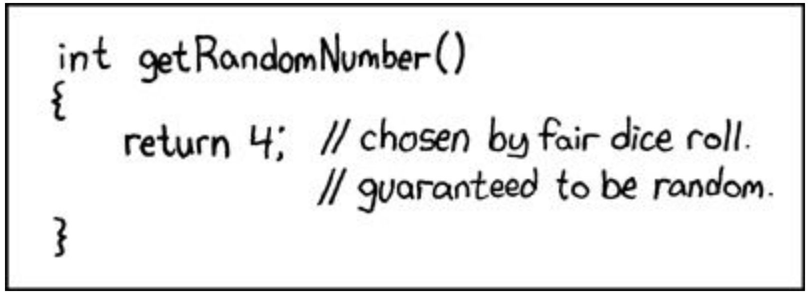
- Many languages have a built-in “random” function

```
unsigned long int next = 1;

/* srand: set seed for rand() */
void srand(unsigned int seed) {
   next = seed;
}

/* rand: return pseudo-random integer on 0..32767 */
int rand(void) {
   next = next * 1103515245 + 12345;
   return (unsigned int)(next/65536) % 32768;
}
```

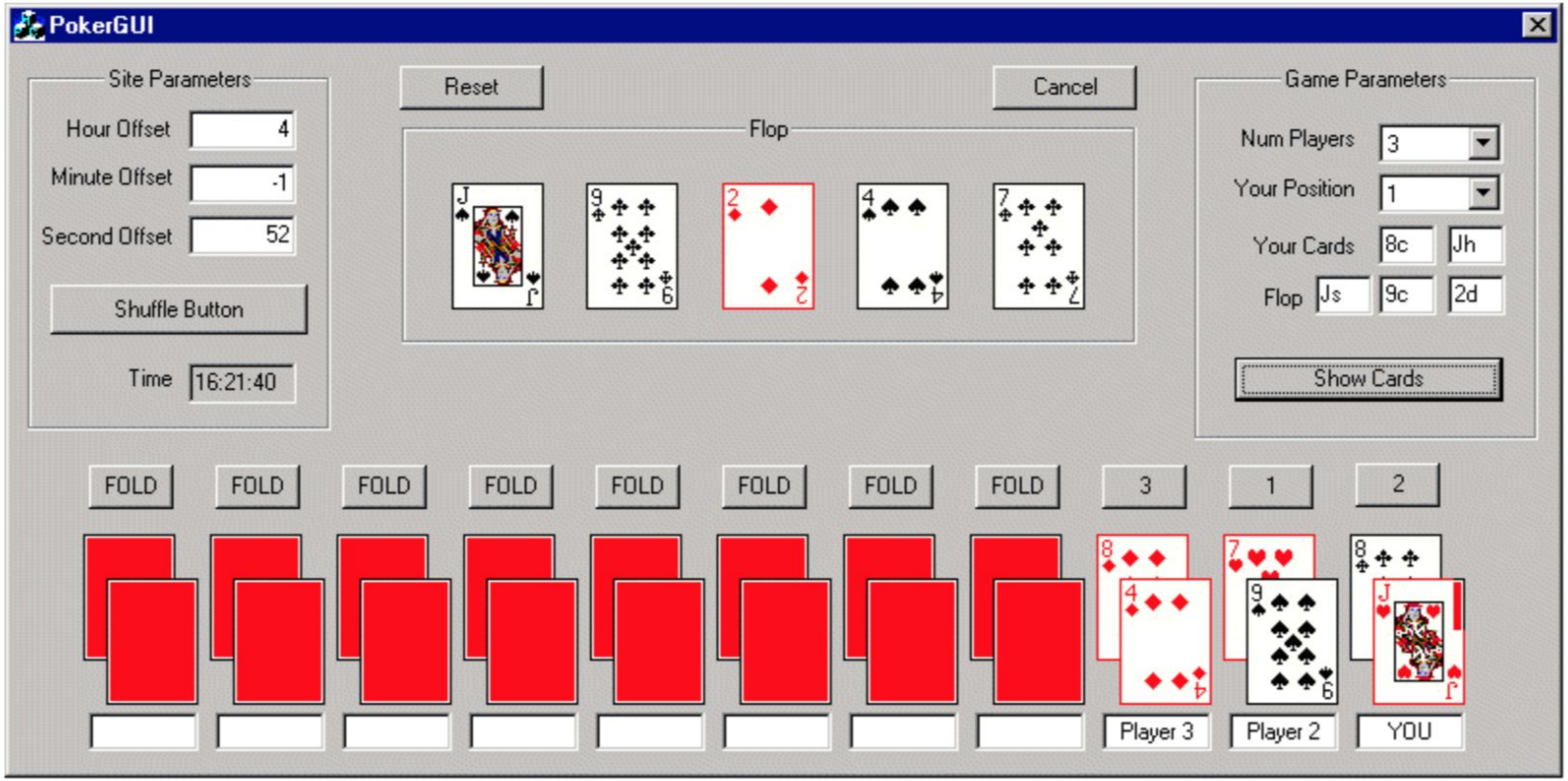
What’s the problem?

Insecure Randomness: C rand()

- Many languages have a built-in “random” function
- Given a few outputs, remaining values are ***predictable***!



https://xkcd.com/221/



More Details

"*How We Learned to Cheat at Online Poker: A Study in Software Security*"

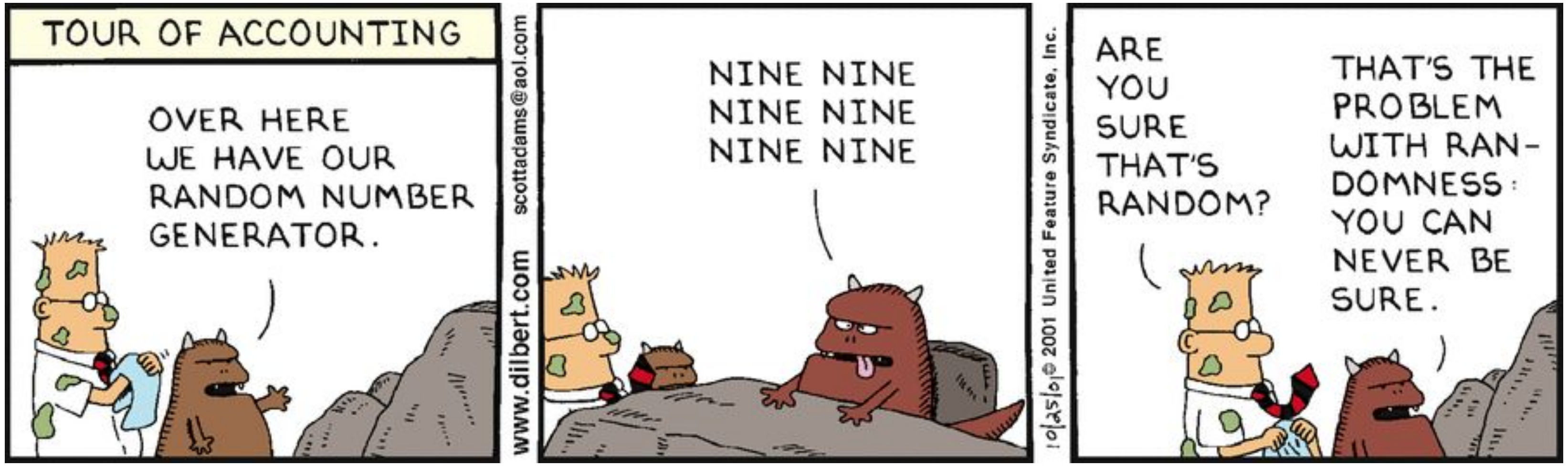
https://www.developer.com/tech/article.php/616221/How-We-Learned-to-Cheat-at-Online-Poker-A-Study-in-Software-Security.htm

Sony PS3 vs. Randomness

- 2010/2011: Hackers found/released ***private root key*** for PS3
- Key used to sign software
  - Load any software on PS3 and execute as "trusted"
  - i.e., Anyone can **pretend to be** Sony
- Flaw: Used same "random" number for every ECDSA signature

More Details
https://events.ccc.de/congress/2010/Fahrplan/attachments/1780_27c3_console_hacking_2010.pdf

So… where does randomness come from?



http://dilbert.com/strip/2001-10-25

Obtaining "True" Randomness

- Gather entropy from unpredictable events

  Ex: Linux "entropy pool" includes mouse & keyboard timing

  – Exposed via
    - /dev/random – **NEVER USE /dev/random – its API is broken and wrong**
    - /dev/urandom – beware of subtle issues with file descriptors and child processes!!!
    - `getrandom` syscall – **always use this syscall when available**
    - Randomness in the Operating System, or How To Keep Evil Children Out Of Your Pool and Other Random Facts – Corrgan-Gibbs and Jana

- Physical random sources (do not use directly!)
  – RDRAND instruction
  – External devices

More fun conversations at: https://lwn.net/Articles/889452/

Quiz Question

Which of the following is likely to consistently provide secure randomness any time you query it?

A. C's `rand()` function

B. /dev/urandom

C. Physical random sources

D. /dev/random

Couple of Reminders from Probability

Probability 101

U: finite set (e.g. U = $\{0,1\}^n$)

Probability distribution P over U is a function P: U ⟶ [0,1] s.t.

$$\sum_{x \in U} P(x) = 1$$

A ⊆ U is called an event and $Pr[A] = \sum_{x \in A} P(x) \in [0,1]$

A random variable is a function X:U⟶V .

X takes values in U and defines a distribution on V

Independence

Definition: events A and B are independent if Pr[A and B] = Pr[A] * Pr[B]

random variables X,Y taking values in V are independent if

$\forall a,b \in V$: Pr[X=a and Y=b] = Pr[X=a] * Pr[Y=b]

Example: $U = \{0,1\}^2 = \{00, 01, 10, 11\}$ and $r \xleftarrow{\$} U$

Define r.v. X and Y as: X = lsb(r) , Y = msb(r)

Pr[X=0 and Y=0] = Pr[r=00] = ¼ = Pr[X=0] * Pr[Y=0]

The Birthday Paradox

In a room of 23 people, the probability that you share a birthday with one other person is greater than 50%.

The Birthday Paradox

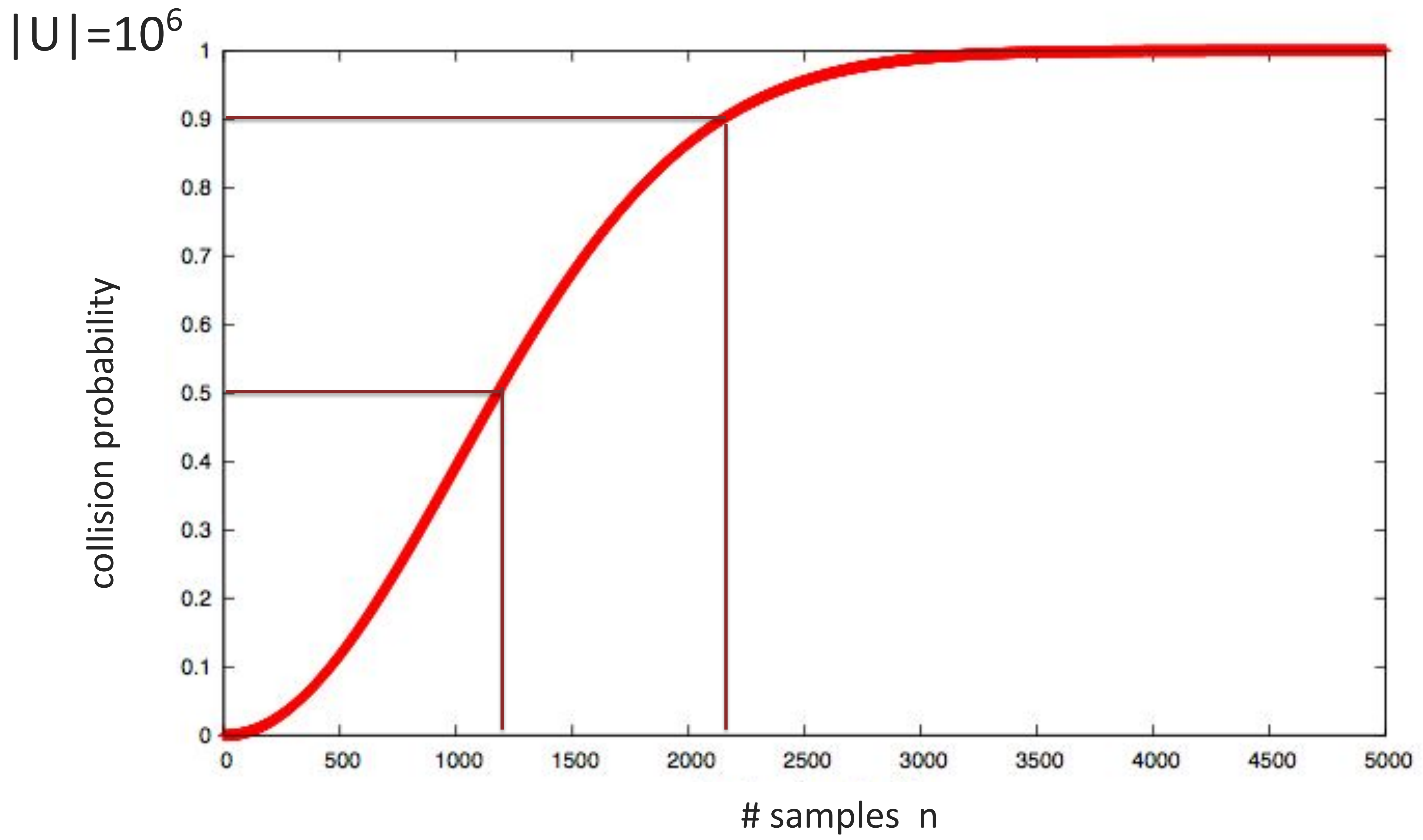
Let $r_1, \ldots, r_n \in U$ be indep. identically distributed random vars.

Theorem: when $n = 1.2 \times |U|^{1/2}$ then $\Pr[\ \exists\, i \neq j:\ r_i = r_j\ ] \geq ½$

Example: Let $U = \{0,1\}^{128}$

After sampling about 2^{64} random messages from U,

some two sampled messages will likely be the same



|U|=10^6
collision probability
samples n
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000

Random Functions and Permutations

Thinking About Mathematical Functions

A function is just a mapping from inputs to outputs:

f_1

x	$f_1(x)$
1	4
2	13
3	12
4	1
5	7

f_2

x	$f_2(x)$
1	1
2	2
3	3
4	4
5	5

f_3

x	$f_3(x)$
1	12
2	3
3	7
4	8
5	10

. . .

~~Which function is *not* random?~~

Thinking About Mathematical Functions

A function is just a mapping from inputs to outputs:

f_1

x	$f_1(x)$
1	4
2	13
3	12
4	1
5	7

f_2

x	$f_2(x)$
1	1
2	2
3	3
4	4
5	5

f_3

x	$f_3(x)$
1	12
2	3
3	7
4	8
5	10

. . .

What is random is the way we *pick* a function

Participation Question

Consider all *functions* of the form F : X -> Y

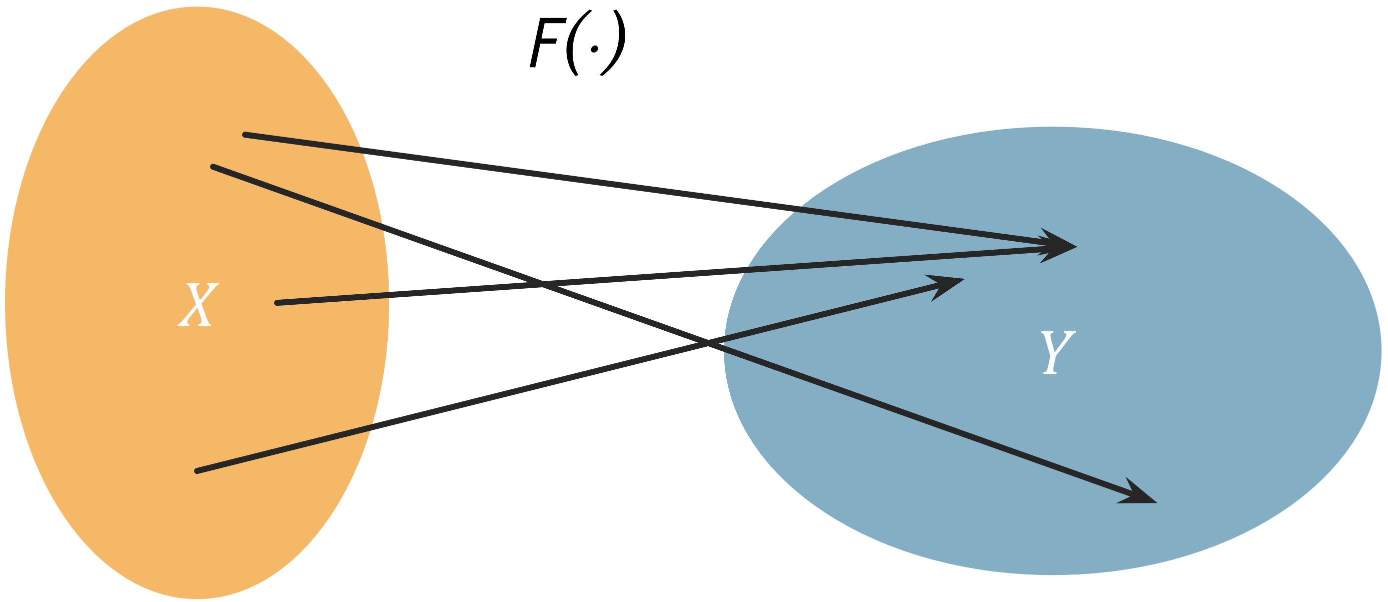
How many possible choices of F are there?

A. |X| * |Y|

B. |X|!

C. $|Y|^{|X|}$

D. $|X|^{|Y|}$



Encryption with Functions

- Alice chooses f: $\{0,1\}^b \rightarrow \{0,1\}^b$ at random from *all possible functions* from $\{0,1\}^b$ to $\{0,1\}^b$
- Alice gives Bob the inverse, f^{-1}
- Given message $m \in \{0,1\}^b$:
  - Alice sends f(m) to Bob
  - Bob decrypts using f^{-1}

Participation Question

Is this a correct cipher?

A. Yes

B. No

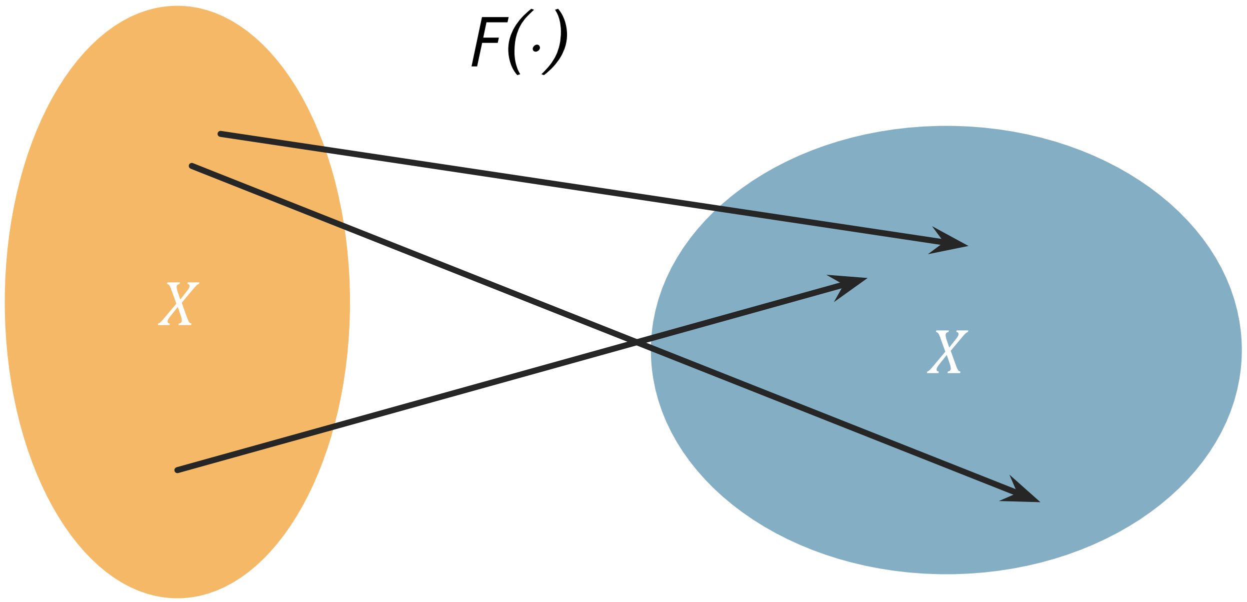
C. I'm not sure

Correctness

$$\forall m \in M, k \in K : D(k, E(k, m)) = m$$

Permutations: Definition

- f : X -> X
- A permutation:
  - Is a function ➔ maps ***every*** element of its domain to ***one*** element of its range
  - Every element in the range is ***mapped to*** by exactly one element of the domain
- In math terms: f is one-to-one
  - $\forall x_1, x_2 \,.\, f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$
- Colloquially, f is a shuffling of X



Participation Question

Consider all ***permutations*** of the form F : X -> X

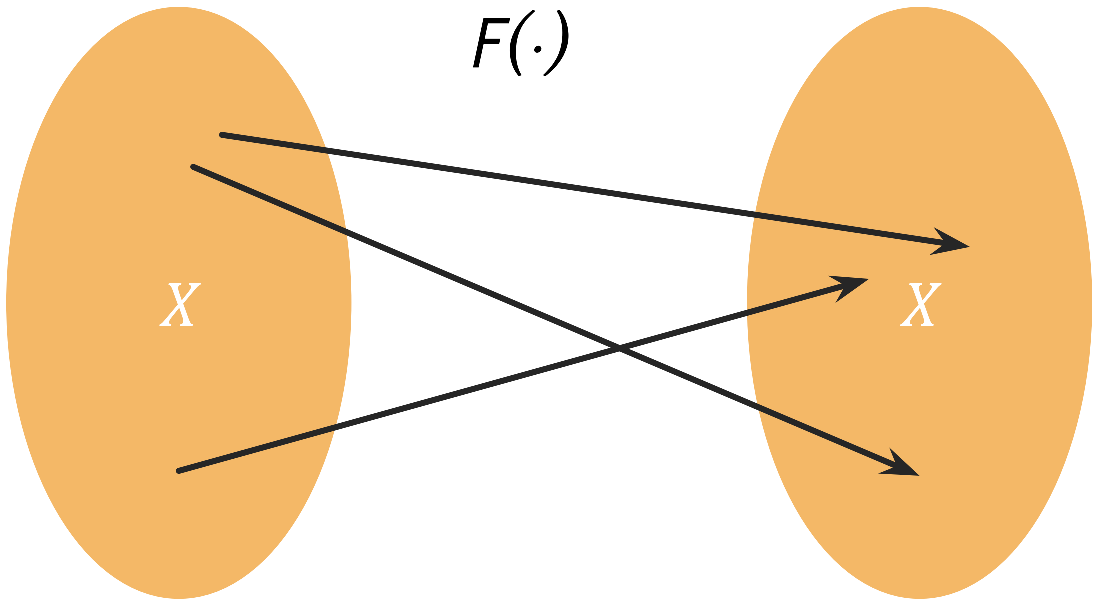
How many possible choices of F are there?

A. 2|X|

B. $|X|^2$

C. $|X|! \approx (\frac{|X|}{e})^{|X|}$

D. $|X|^{|X|}$



Better Encryption Scheme?

- Alice chooses f: $\{0,1\}^b$ -> $\{0,1\}^b$ at random from all possible ***permutations*** from $\{0,1\}^b$ to $\{0,1\}^b$
- Alice gives Bob the inverse, f^{-1}
- Given message $m \in \{0,1\}^b$:
  - Alice sends f(m) to Bob
  - Bob decrypts using f^{-1}

<u>Participation Question</u>

Is this a correct cipher?

A. Yes

B. No

C. I'm not sure

Good cipher?

Better Encryption Scheme?

- Alice chooses f: $\{0,1\}^b$ -> $\{0,1\}^b$ at random from all possible ***permutations*** from $\{0,1\}^b$ to $\{0,1\}^b$
- Alice gives Bob the inverse, f^{-1}
- Given message $m \in \{0,1\}^b$:
  - Alice sends f(m) to Bob
  - Bob decrypts using f^{-1}

Did we bypass "bad news" theorem?

Ευχαριστώ και καλή μέρα εύχομαι!

Keep hacking!