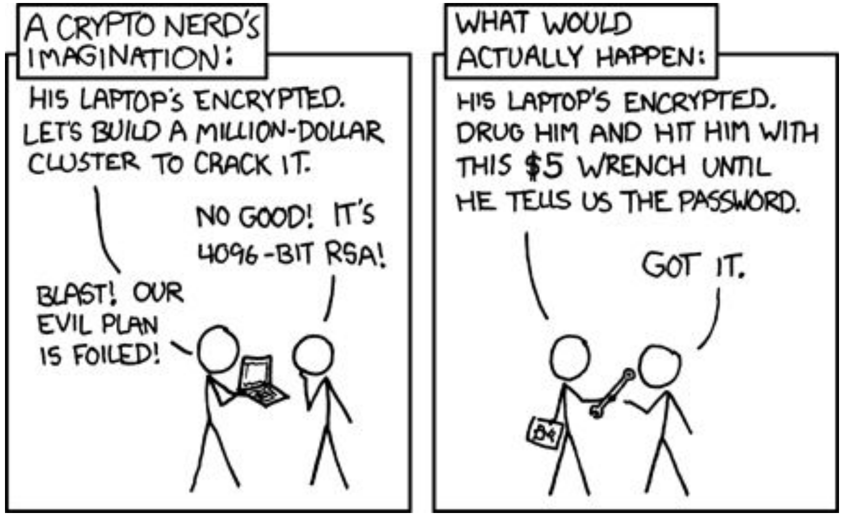
Διάλεξη #12 - Introduction to Cryptography

Εθνικό και Καποδιστριακό Πανεπιστήμιο Αθηνών

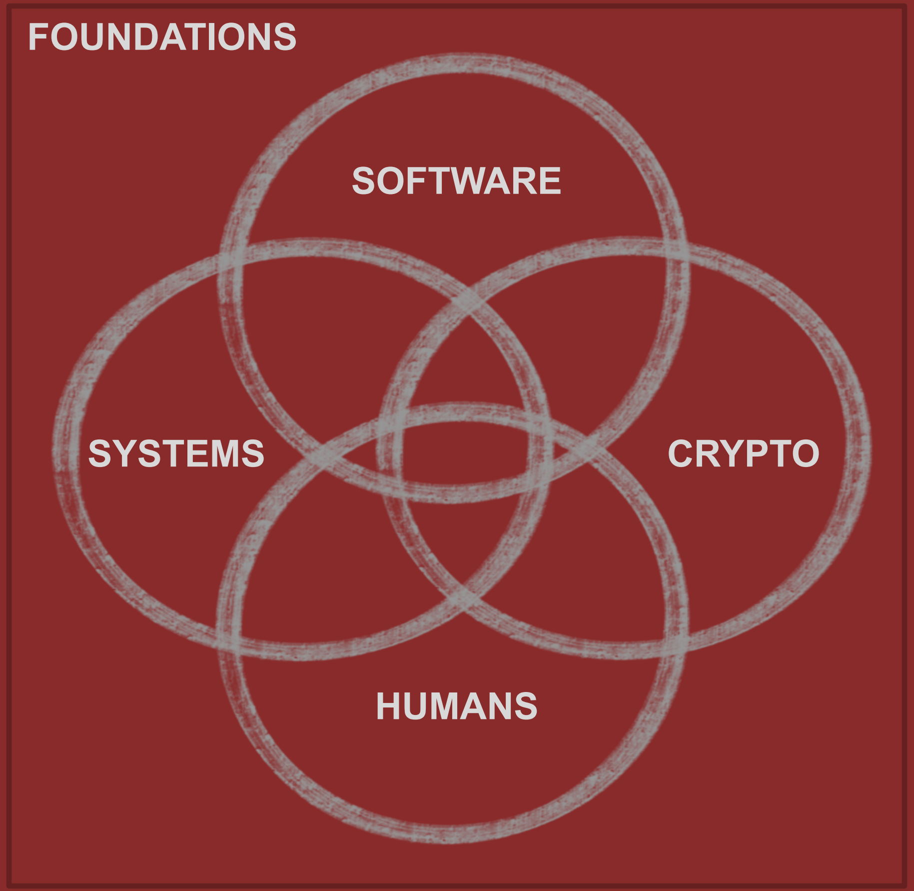
Εισαγωγή στην Ασφάλεια

Θανάσης Αυγερινός



Hfr frpher pelcgb cyrnfr!

Huge thank you to David Brumley from Carnegie Mellon University for the guidance and content input while developing this class (some slides from Dan Boneh @ Stanford!)



Ανακοινώσεις / Διευκρινίσεις

- Ουδέν νεότερο!

Την Προηγούμενη Φορά

- CFG and call graph definitions
- Insensitive and sensitive program analysis types
  - Soundness / Unsoundness
- Type safety

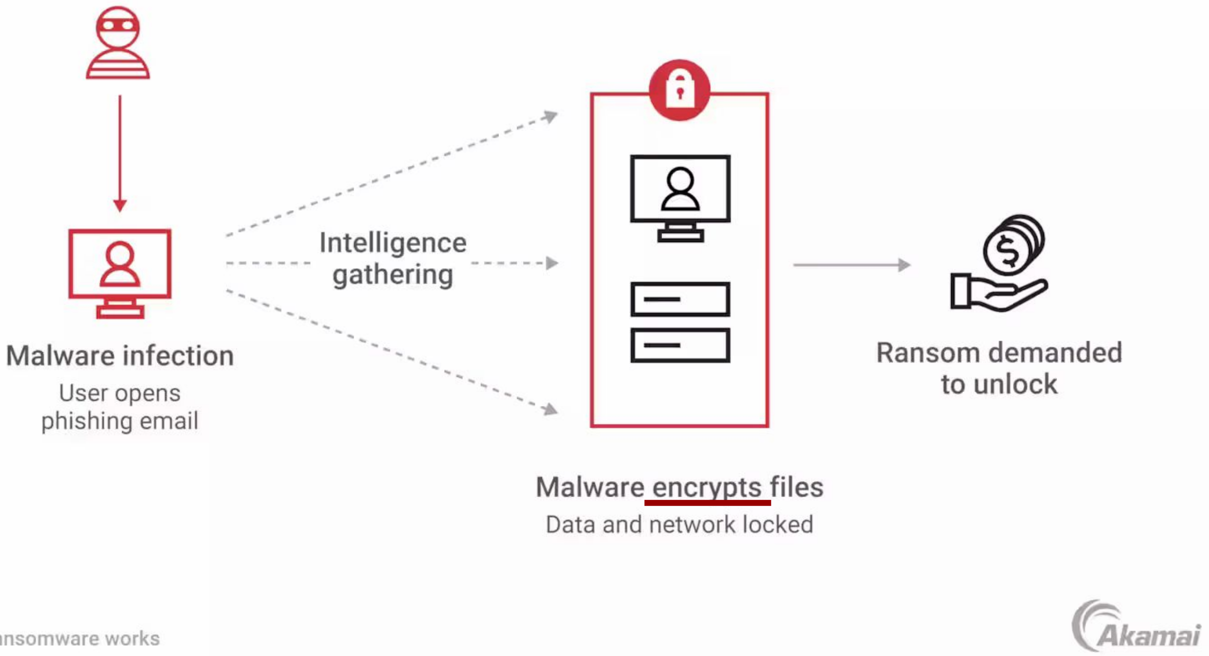
Security in the News

PipeMagic Trojan Exploits Windows Zero-Day Vulnerability to Deploy Ransomware

Apr 09, 2025 Ravie Lakshmanan

The vulnerability in question is CVE-2025-29824, a privilege escalation bug in CLFS exploited to achieve SYSTEM privileges. It was fixed by Redmond as part of its Pat for April 2025.

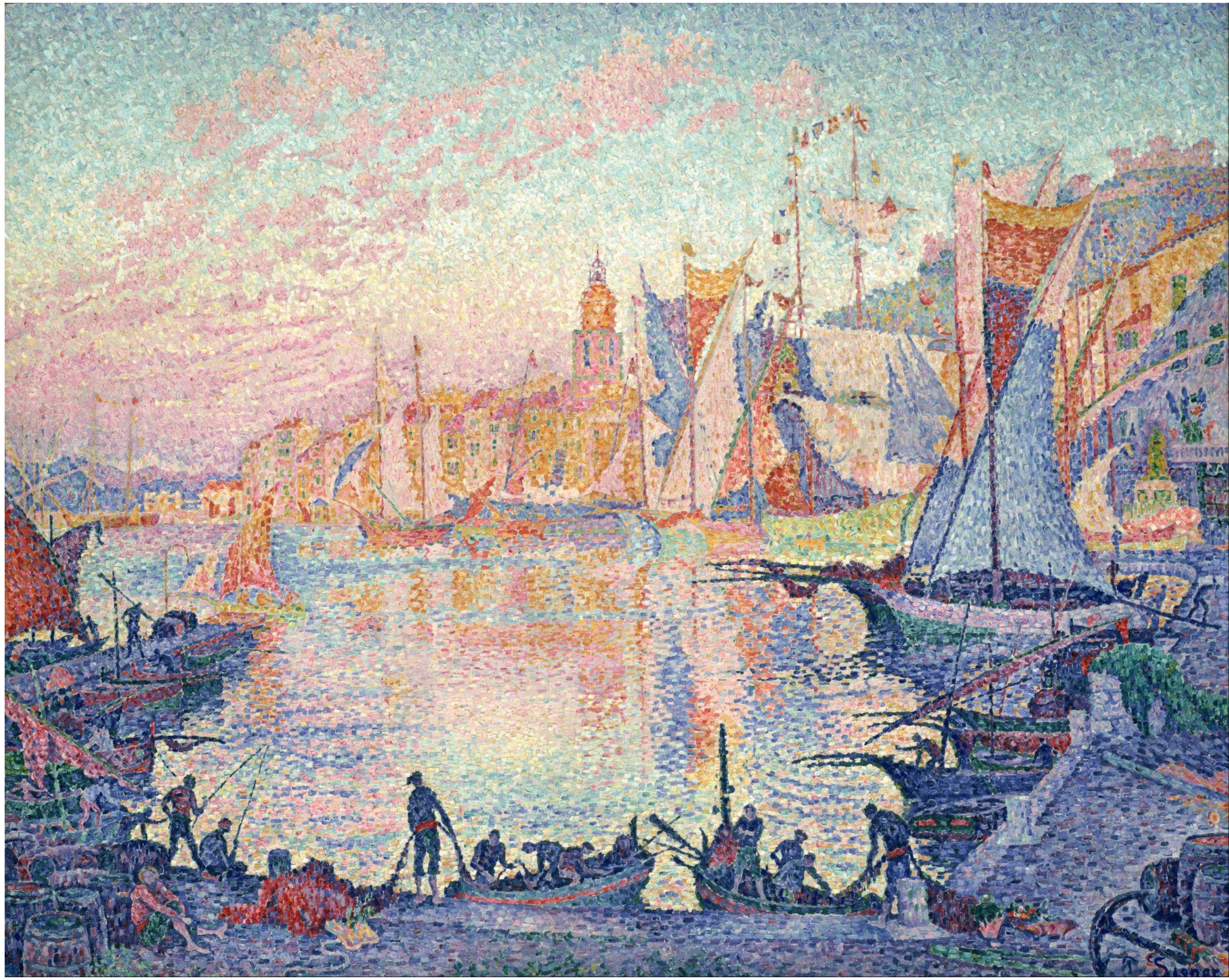
Microsoft is tracking the activity and the post-compromise exploitation of CVE-202 moniker Storm-2460, with the threat actors also leveraging a malware named Pipel exploit as well as ransomware payloads.



How ransomware works

Σήμερα

- About cryptography
- Terminology
- Traditional ciphers
- One-time pad

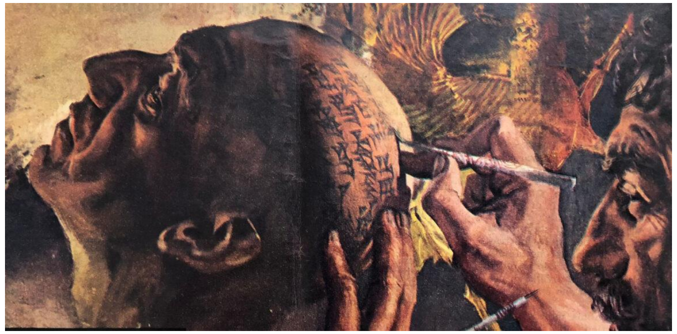


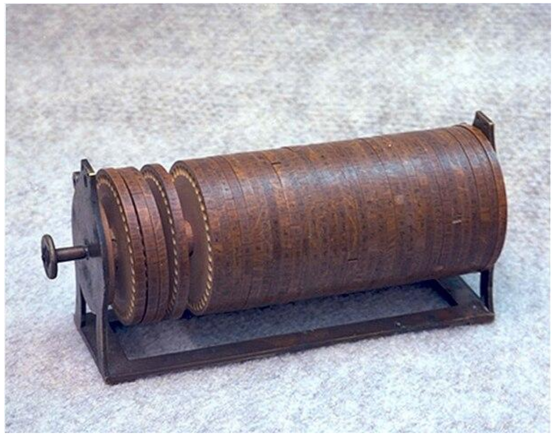
About Cryptography (aka crypto)

Cryptography

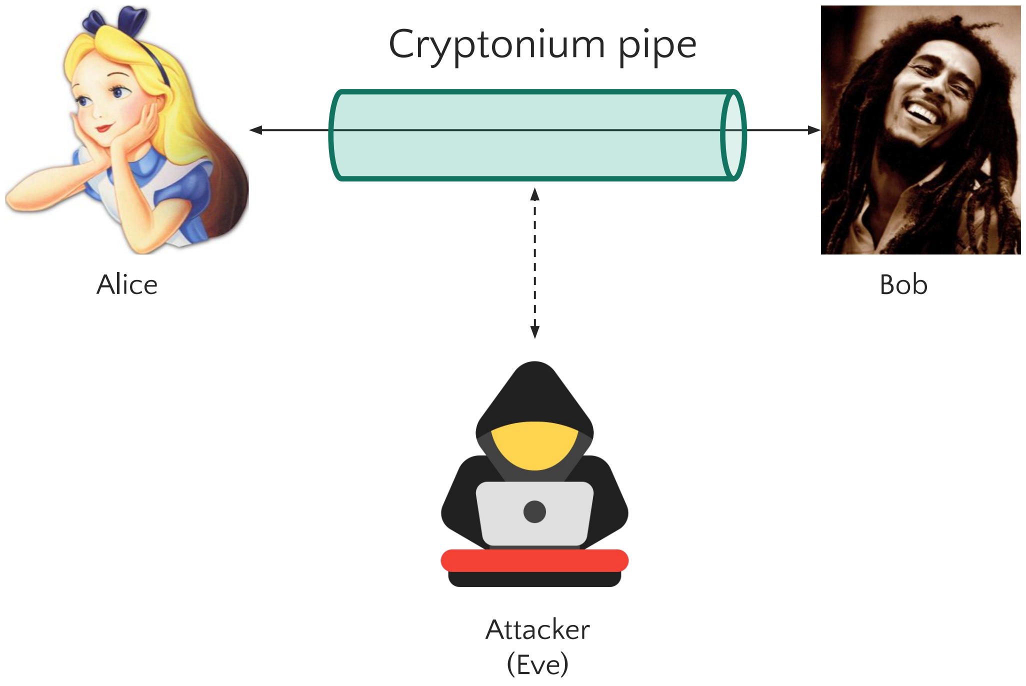
Cryptography, or **cryptology** (from Ancient Greek: κρυπτός, romanized: *kryptós* "hidden, secret"; and γράφειν *graphein*, "to write", or -λογία *-logia*, "study", respectively[1]), is the practice and study of techniques for secure communication in the presence of adversarial behavior.[2] More generally, cryptography is about constructing and analyzing protocols that prevent third parties or the public from reading private messages.[3]

https://en.wikipedia.org/wiki/Cryptography





Cryptography is About Secure Communication



Do we remember the 4 security properties?

Confidentiality (Secrecy), Integrity, Authenticity and Availability

Computer Security ≠ Crypto

"Those who think that cryptography can solve their problems don't understand cryptography and don't understand their problems." – Bruce Schneier

- How can we generate good keys?
- How do we know the crypto implementations are correct?
- How do we build networks that are secure ***and*** available?
- How do we ensure only Alice can access her keys?
- …

Cryptography is everywhere…

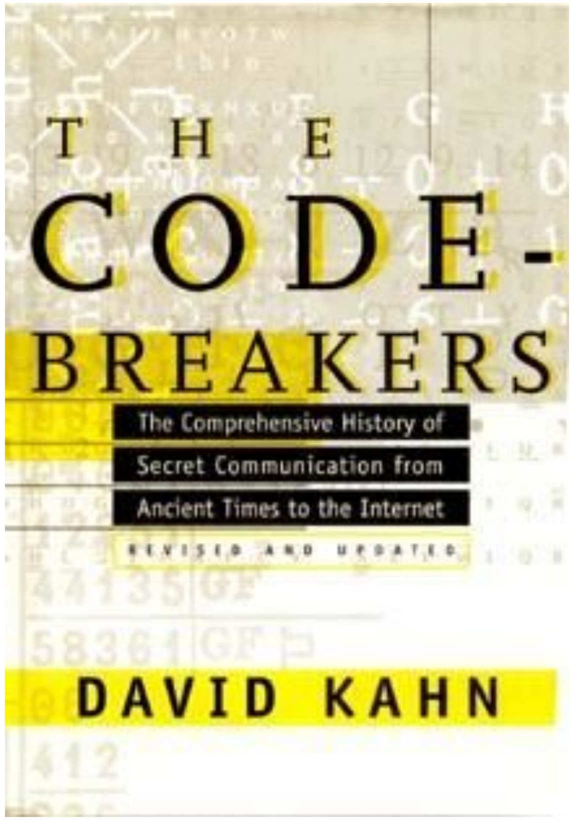
- Secure communication:
  - HTTPS, 802.11i WPA2 (and WEP), SSL, GSM, Bluetooth
- Encrypting data at rest:
  - BitLocker (Windows), FileVault (MacOS)
- Content protection:
  - CSS (DVD), AACS (Blue-Ray)
- User authentication
  - Kerberos, HTTP Digest
- Crypto currencies:
  - Bitcoin, Ethereum, etc.
- Spectrum security
  - Frequency Hopping Spread Spectrum

… and much, much more

…but how did it get there?

David Kahn,
“The Code-Breakers”

(Revised ed. 1996)

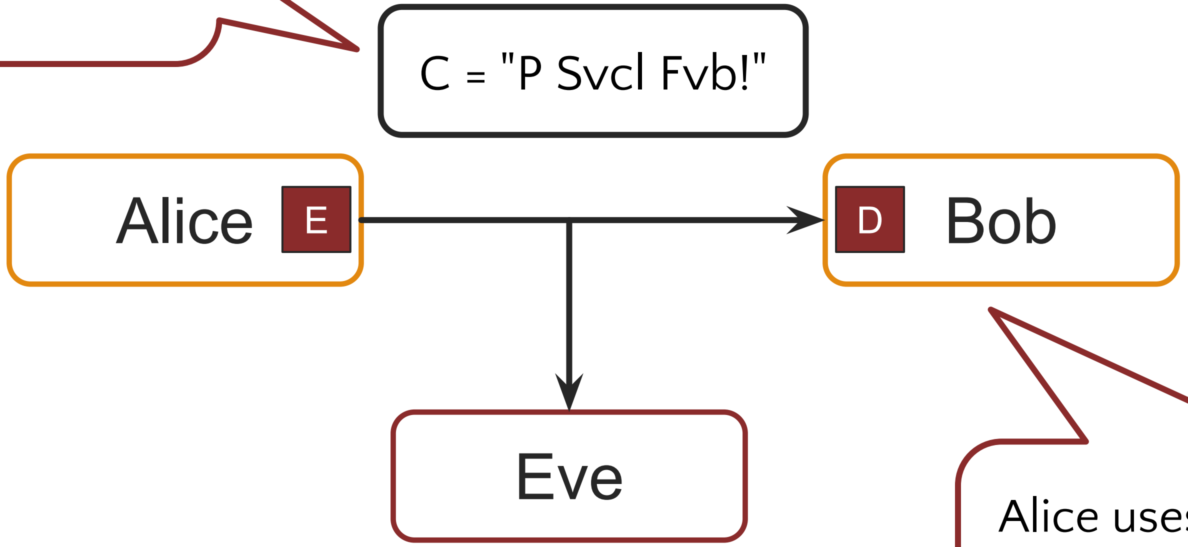


It started with Secrets

The Secrecy Game

Alice uses an *encryption* algorithm E to generate a *ciphertext* C (C = E(M)). C is visible to Eve.

M = "I Love You!"



M = "I Love You!"

Alice wants to send a message M – the *plaintext* to Bob without Eve knowing what was sent

Alice uses a *decryption* algorithm D to recover the original plaintext M (M = D(C)).

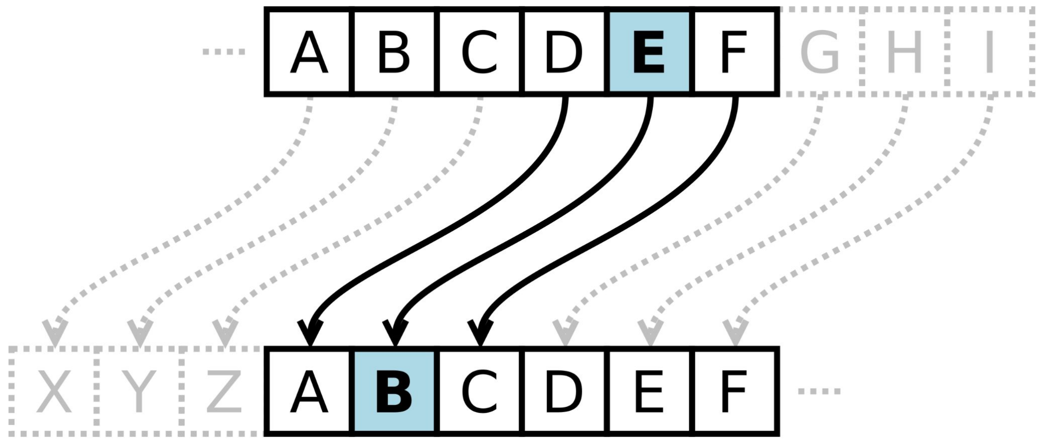
An algorithm that allows the encryption and decryption (E, D) of data is called a *cipher*. Typically requires a secret in order to function, also known as the *key* K.

Caesar Cipher:
replace each letter with letter + 3 (e.g., A →D)

A B C D E F G H I J

K L M N O P Q R

S T U V W X Y Z



How large is the key space?

How would you *attack* it?

Popular variant:
Rot-13


Julius Caesar
100 BC- 44 BC

Substitution Cipher

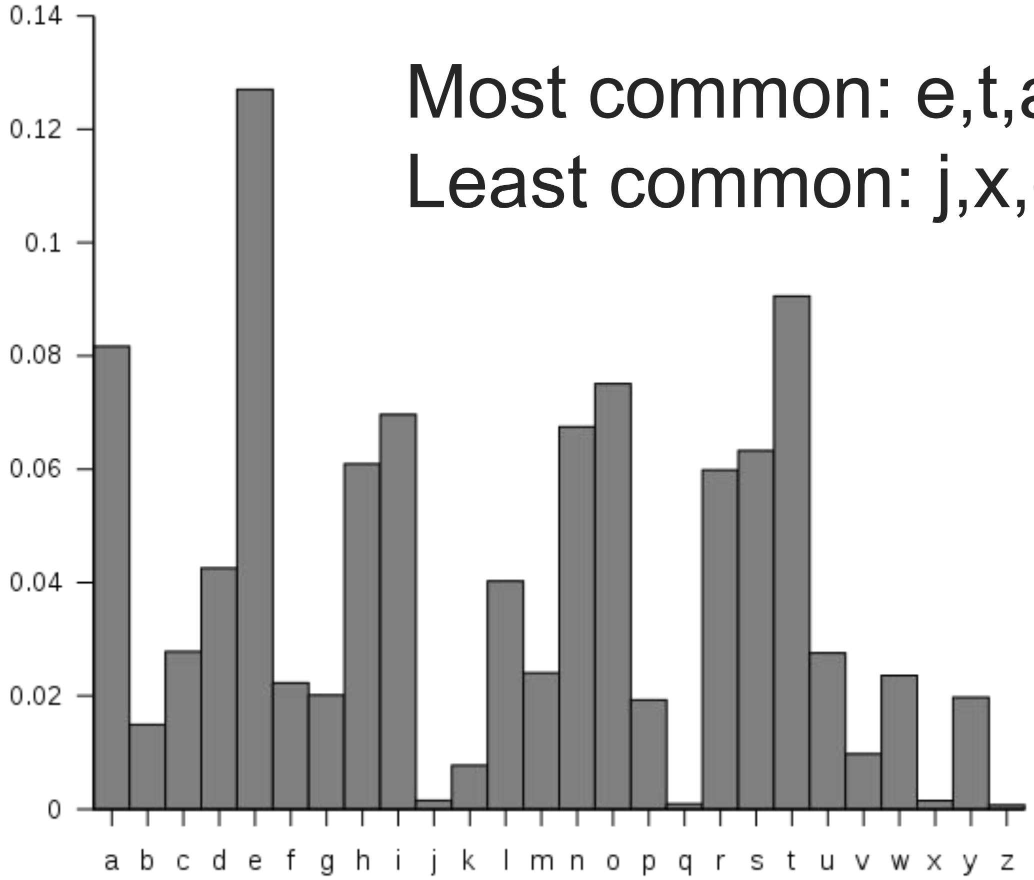
- Pick a random permutation from [A-Z] -> [A-Z]
- How large is the key space?
- How do you remember the permutation?
- Example: Use a secret key word/phrase
  - Map: `ABCDEFGHIJKLMNOPQRSTUVWXYZ`
    to: `MONKEYSABCDFGHIJLPQRTUVWXY`
- "State of the art" for thousands of years
- How large is this key space?

How would ***<u>you</u>*** decrypt messages encrypted with a substitution cipher?

Attacking Substitution Ciphers

Trick 1:
Word Frequency

Trick 2:
Letter Frequency

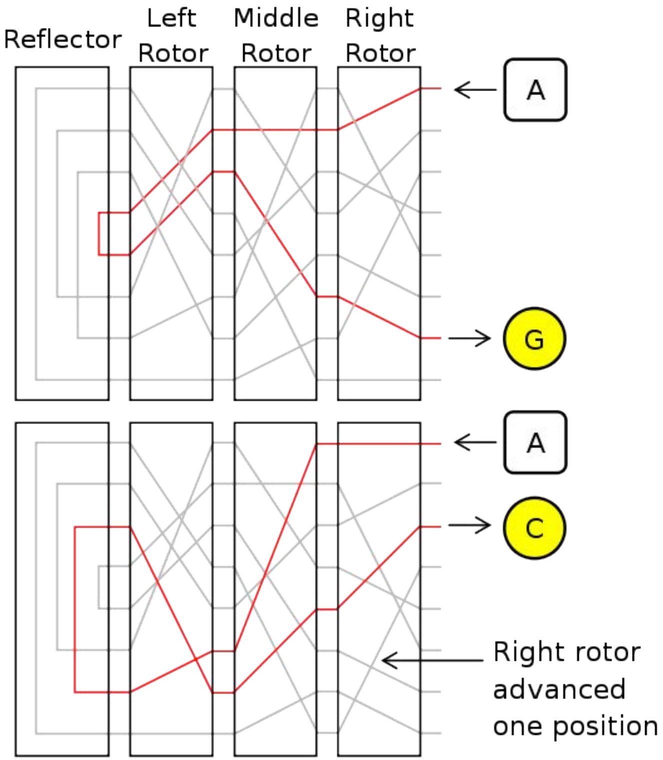


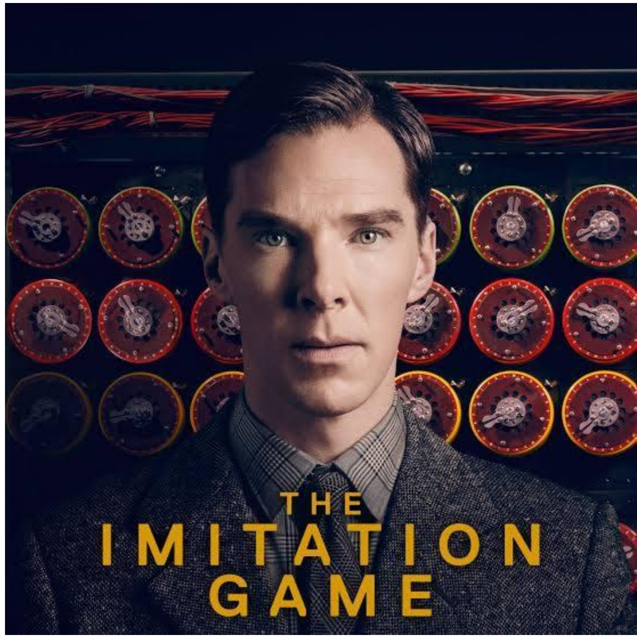
Most common: e,t,a,o,i,n
Least common: j,x,q,z

image source: wikipedia

Rotor Machines

Most famous: the Enigma (3–5 rotors)





Used a substitution cipher along with other complexities on top.

Employed by Nazi Germany during World War II.

https://picoctf.com

Jvl mlwclk yr jvl owmwez twp yusl w zyduo

pjdcluj mqil zydkplmr. Hdj jvlz tykilc vwkc jy

mlwku jvl wkj yr vwsiquo, tvqsv vlmflc mlwc

jvlg jy oklwjulpp. Zyd vwnl jvl fyjlujqwm jy cy

jvl pwgl. Zydk plsklj fwpptykc qp: JYWPJ

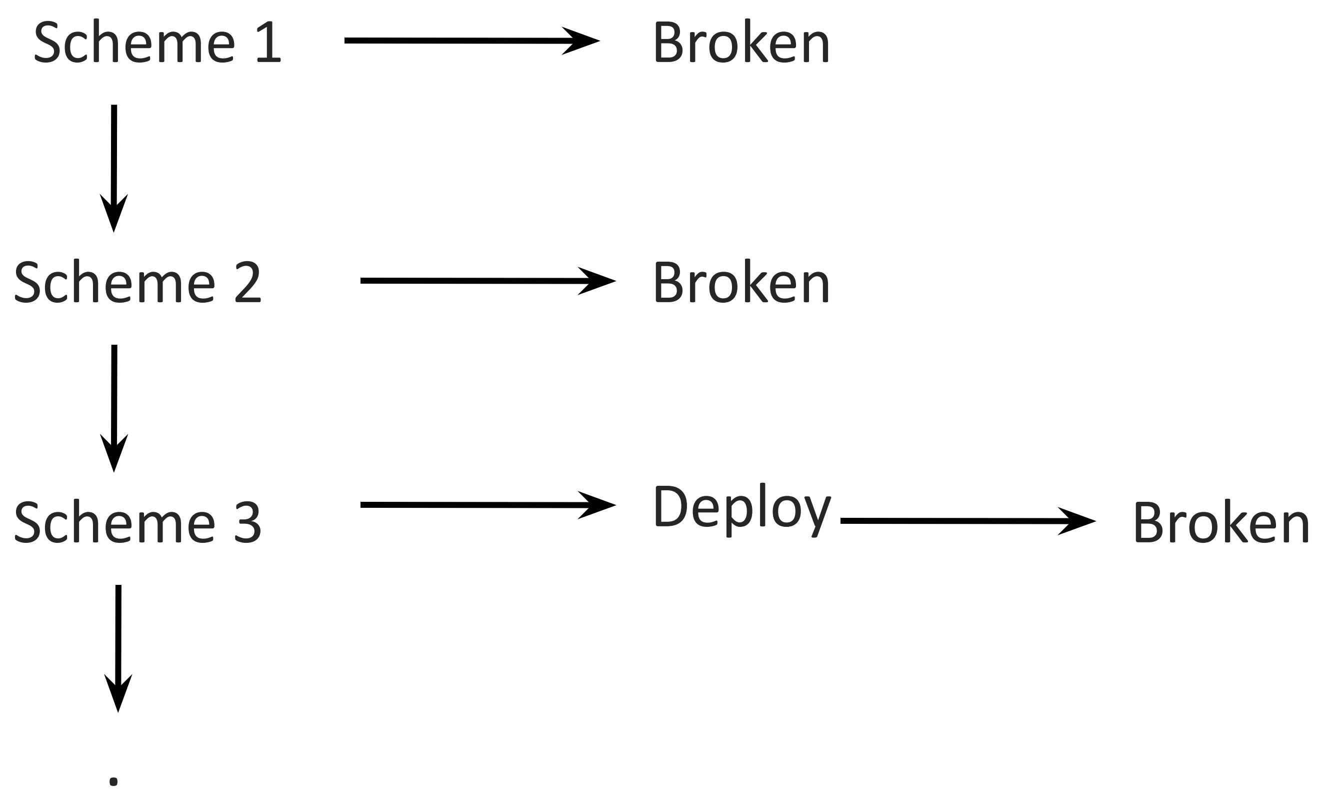
https://cryptopals.com

Kerckhoffs' Principle

A cryptosystem should be secure, even if *everything* about the system, *except* the key, is public knowledge.

Does this remind us of a standard security principle?

Classical Approach: Iterated Design



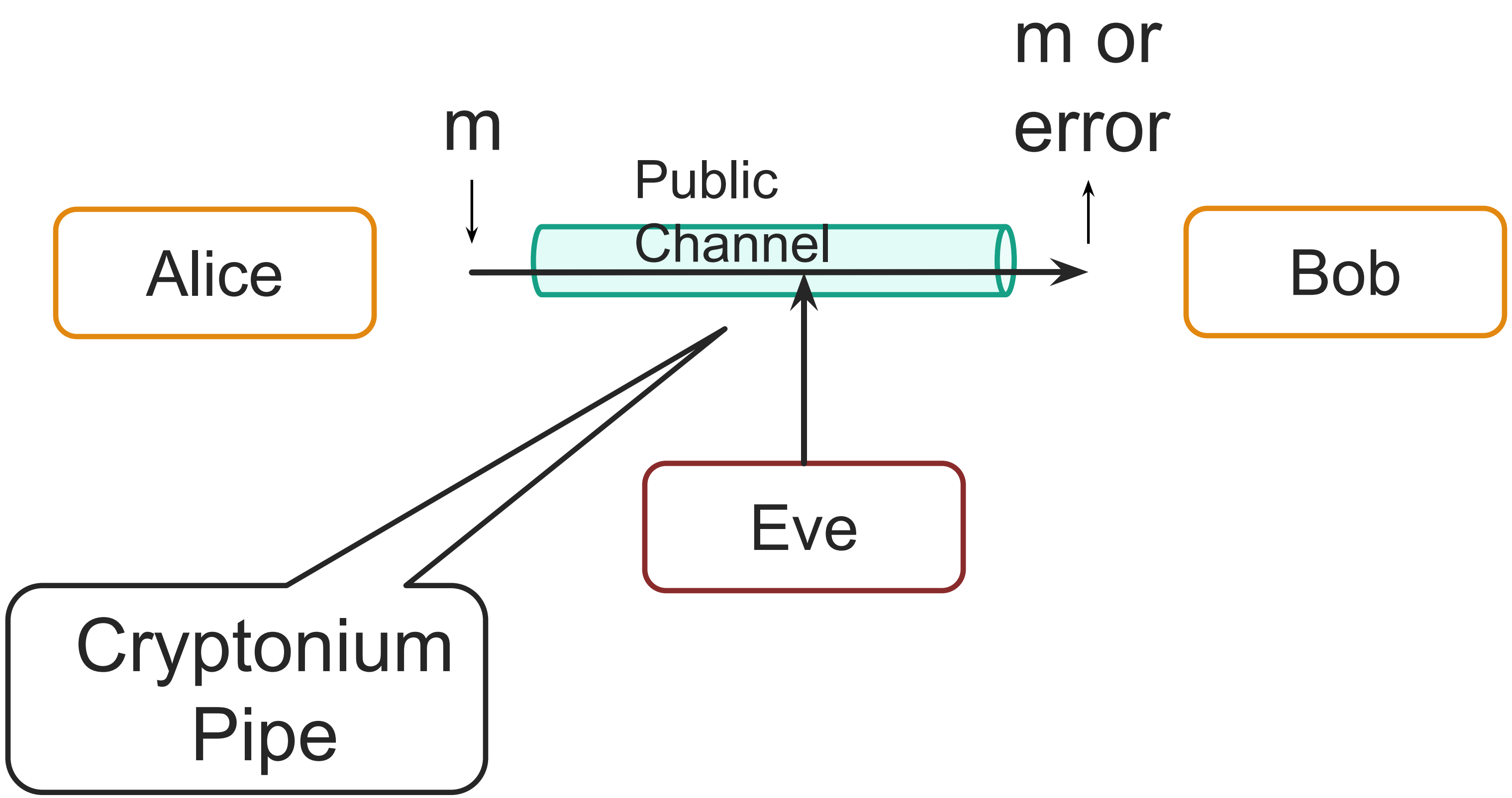
No way to say anything is secure
(and you may not know when broken)

Iterated design was all we knew until 1945

Claude Shannon: 1916 - 2001

- Formally define:
  - *security goals*
  - *adversarial models*
  - *security of system w.r.t. goals*
- Beyond iterated design: Proof!

Our Goal: Secure Channel

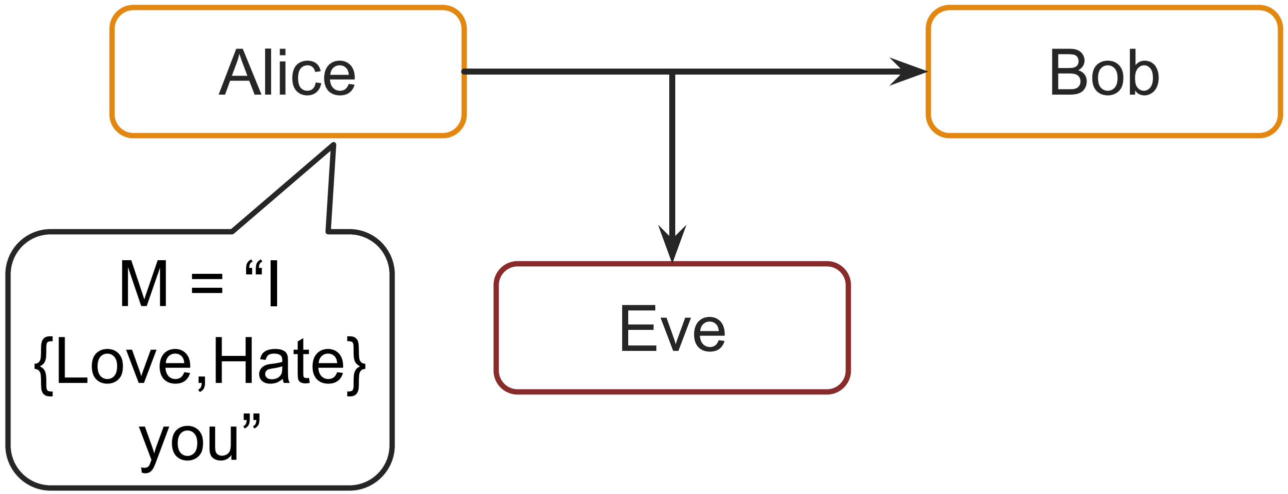


Sub Goal 1: *Secrecy*

Eve should not be able to learn m

How Secure Is Secure Enough?

Suppose there are two possible messages that differ on one bit, e.g., whether Alice Loves or Hates Bob



Secrecy means Eve still should not be able to determine which message was sent

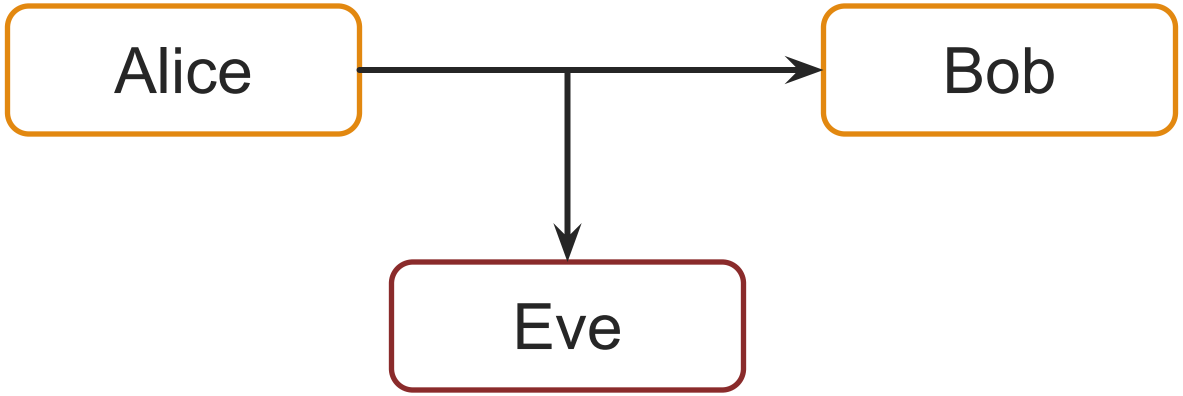
Security guarantees should hold for all messages, not just a particular kind of message

Secure Against Whom (Adversary)?

<u>Eve's Possible Powers</u>

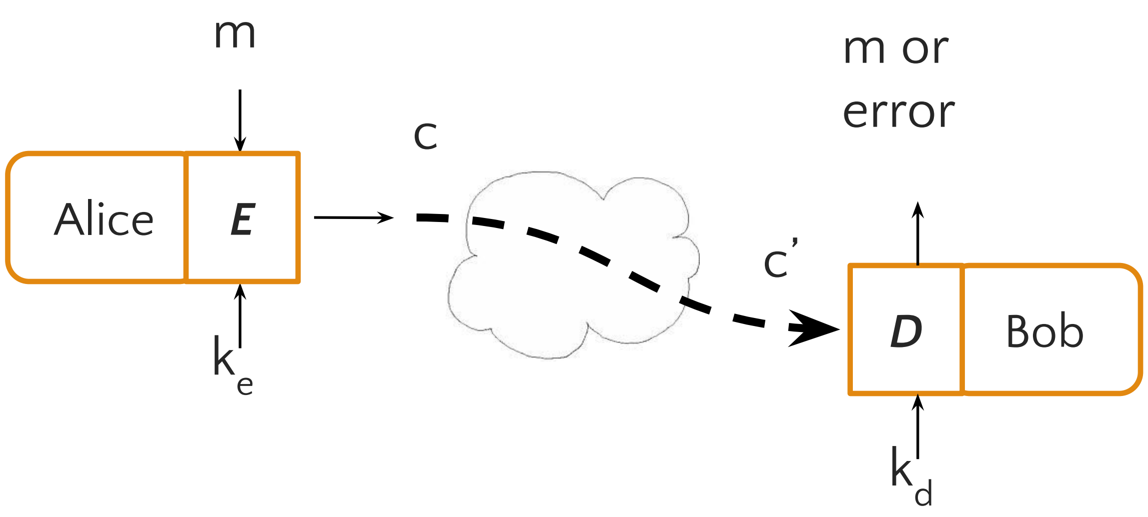
- *<u>Ciphertext only</u>*: only access to ciphertext
- *<u>Known Plaintext Attack (KPA)</u>*: Access to a <message,ciphertext> list
- *<u>Chosen Plaintext Attack (CPA)</u>*: Ability to have messages encrypted
- *<u>Chosen Ciphertext Attack (CCA):</u>* Ability to have ciphertexts decrypted

Note: Eve succeeds only if she gains information on a "fresh" ciphertext

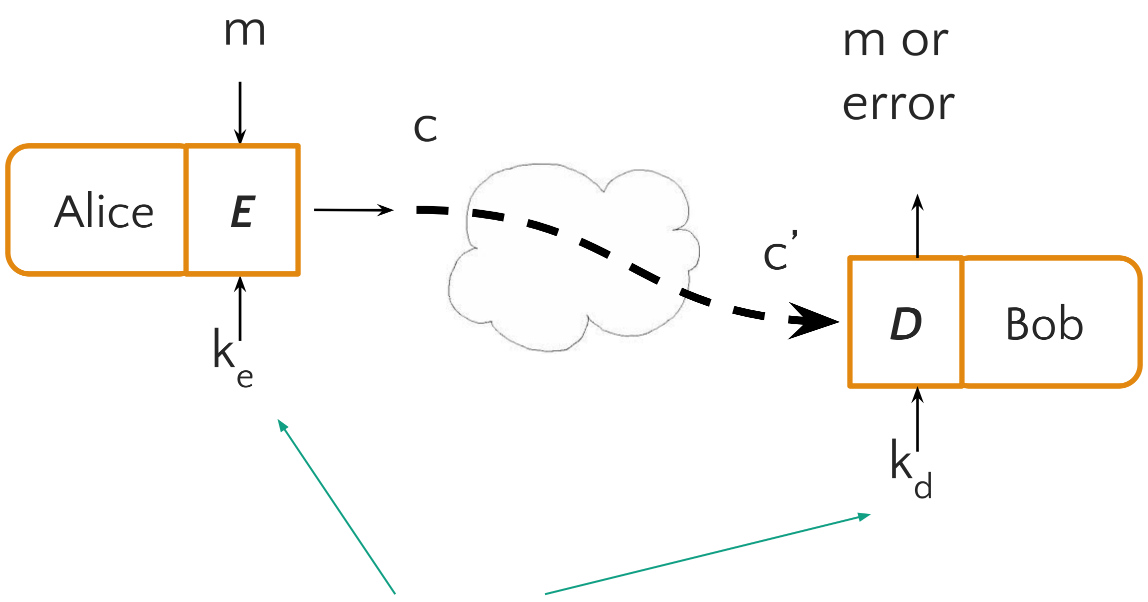


Generic Encryption Scheme

Var	Description
m	Message (aka plaintext). From the message space M
c	Ciphertext. From the ciphertext space C
E	Encryption Algorithm
D	Decryption Algorithm
k_e	Encryption key from the key space K
k_d	Decryption key from the key space K

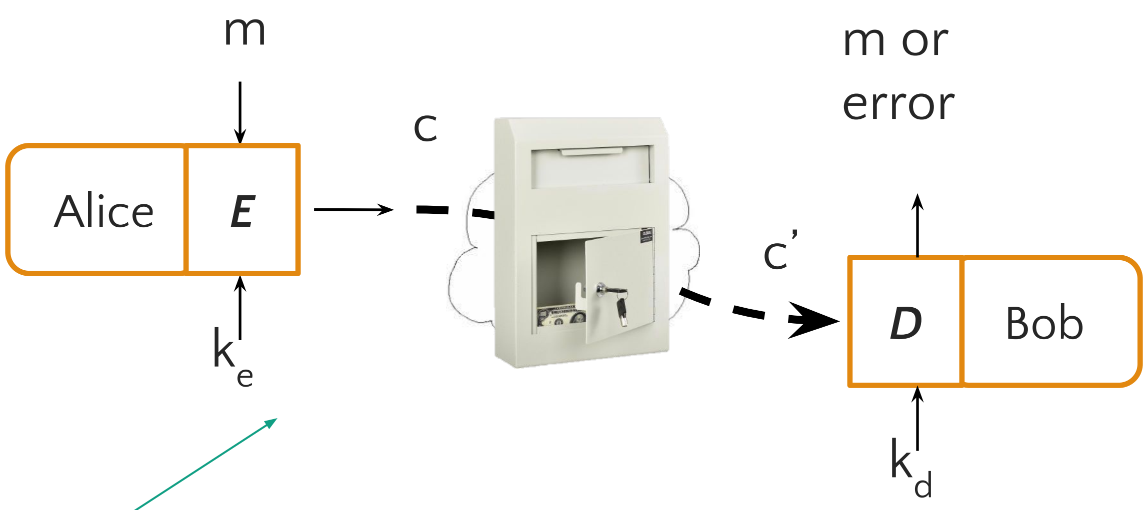


Symmetric Encryption



- $k = k_e = k_d$
- Everyone who knows k knows the full secret

Asymmetric Encryption



- k_e != k_d
- Encryption Example:
  - Bob generates private (k_d)/public(k_e) keypair
    - Sends Alice public key
  - To encrypt a message to Bob, Alice computes $c = E(m,k_e)$
  - To decrypt, Bob computes $m = D(c', k_d)$

An Interesting Story...

1974

- A student enrolls in the Computer Security course @ Stanford
- Proposes idea for public key crypto
- Professor shoots it down

Picture: http://www.merkle.com

1975

- Submits a paper to the Communications of the ACM

- “I am sorry to have to inform you that the paper is not in the main stream of present cryptography thinking and I would not recommend that it be published in the Communications of the ACM. Experience shows that it is extremely dangerous to transmit key information in the clear."

Picture: http://www.merkle.com

Today

Ralph Merkle:
A Pioneer of Cryptography

Picture: http://www.merkle.com

Intermission

Interview Question

You are given an array of N integers where all of them repeat exactly twice except for one that is unique. Describe an efficient time and space algorithm to find the unique element.

Two Key Properties to Remember

Self-inverse: $x \oplus x = 0$

Identity: $x \oplus 0 = x$

Ciphers and the one-time pad

Symmetric Cipher

Def'n: A **symmetric cipher** over key space $\mathscr{K}$, message space $\mathscr{M}$, and ciphertext space $\mathscr{C}$ comprises three polynomial time algorithms:

1. *KeyGen*($\lambda \in \mathbb{N}$) $\rightarrow$ k $\in \mathscr{K}$
   A randomized algorithm that returns a fresh key of length λ. We say that λ is the *security parameter.*

2. *E*(k $\in \mathscr{K}$, m $\in \mathscr{M}$) $\rightarrow$ c $\in \mathscr{C}$
   A (usually randomized) algorithm that encrypts *m* under *k* to produce ciphertext *c.*

3. *D*(k $\in \mathscr{K}$, c $\in \mathscr{C}$) $\rightarrow$ m $\in \mathscr{M}$ U ERROR
   A *deterministic* algorithm that decrypts c with key k, returning either a message *m* or an ERROR indicating decryption failure.

We say that a cipher is ***correct*** if it satisfies the following condition:

$$\forall\ k \in \mathscr{K} : \forall\ m \in \mathscr{M} : D(k, E(k, m)) = m$$

The One-Time Pad (OTP)

Miller, 1882 and Vernam, 1917

$$E(k,m) = k \oplus m = c$$

$$D(k,c) = k \oplus c = m$$

$\mathcal{K} = \mathcal{M} = \mathcal{C} = \{0,1\}^n$

$$\begin{array}{cccccccc} & \text{m:} & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ \oplus & \text{k:} & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ \hline & \text{c:} & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ \oplus & \text{k:} & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ \hline & \text{m:} & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{array}$$

The One-Time Pad (OTP)

Miller, 1882 and Vernam, 1917

$$E(k, m) = k \oplus m = c$$

$$D(k, c) = k \oplus c = m$$

Is it a cipher?

✔ Efficient

✔ Correct

$$\begin{aligned} D(k, E(k, m)) &= D(k, k \oplus m) \\ &= k \oplus (k \oplus m) \\ &= 0 \oplus m \\ &= m \end{aligned}$$

Participation Question

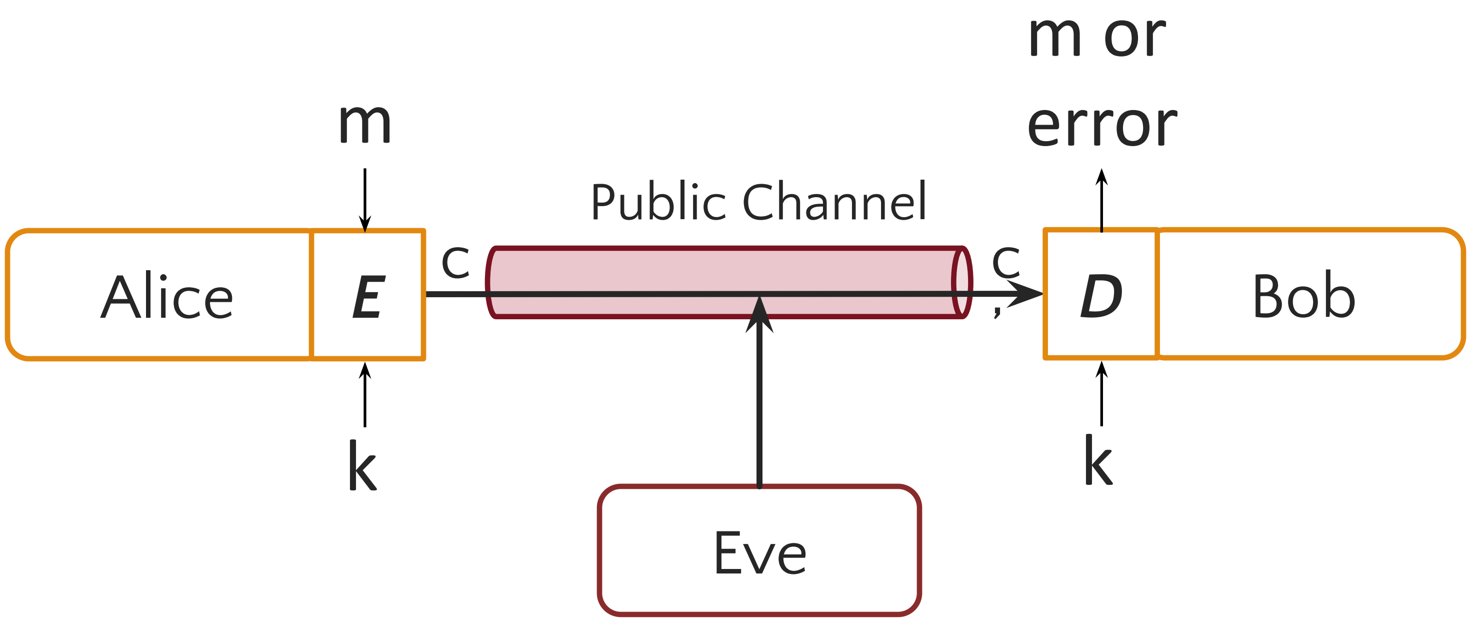
Given *m* and *c* encrypted with an OTP, can you compute the key?

$$E(k, m) = k \oplus m = c$$

1. No
2. Yes, the key is k = m ⊕ c
3. I can only compute half the bits
4. Yes, the key is k = m ⊕ m

So is OTP secure?

Our Goal: Secure Communication



Sub Goal 1: _Secrecy_
Eve should not be able to learn m.

Possible Security Definition

- Given a message space M
- Given a ciphertext c = Enc(k, m) for m in M
- We want Pr[Adversary guesses m] <= 1/|M|

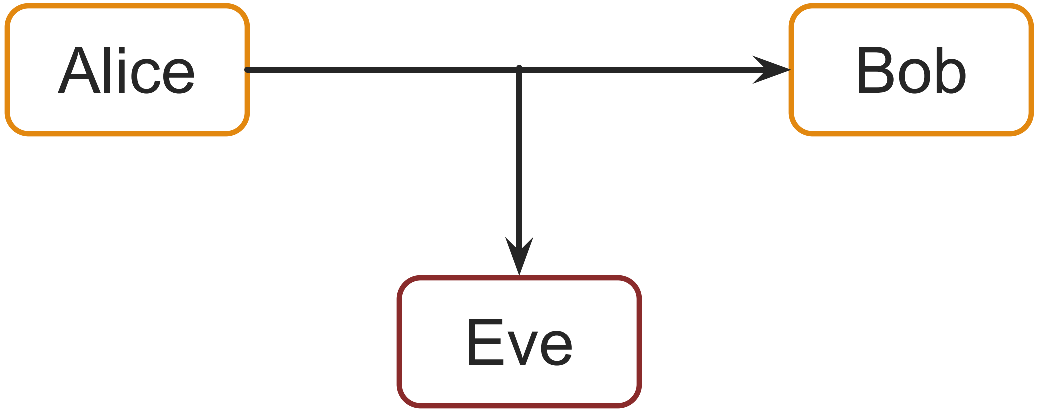
Is this a good definition of security?

A. Yes
B. No
C. I don't know

Sadly, *no cipher* can be secure by that definition!

Suppose Eve knows that there are exactly 3 messages Alice may send *and* the probability of each:

- m_1: The attack is at 12pm. The probability of this message is 1/2
- m_2: The attack is at 3pm. The probability of this message is 1/4
- m_3: The attack is at 5pm. The probability of this message is 1/4



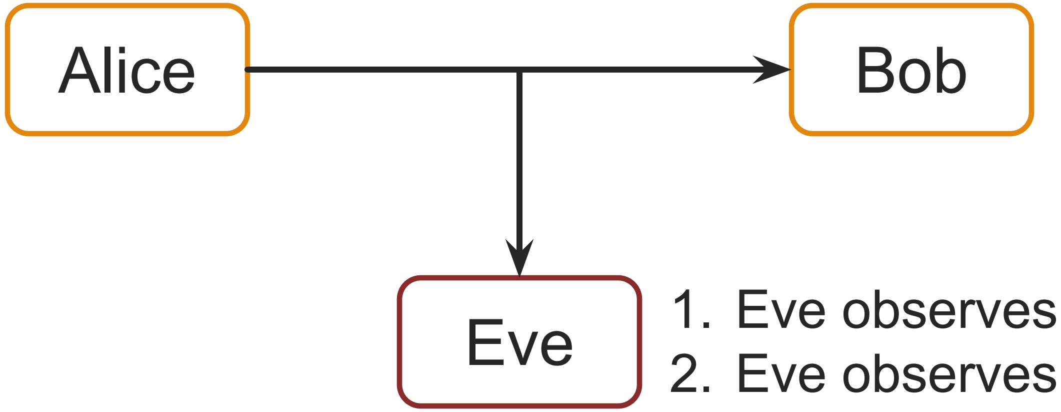
Always guess m_1

M	m_1	m_2	m_3
Pr[M=m]	½	¼	¼

Perfect Secrecy [Shannon1945]
(Information Theoretic Secrecy)

<u>Defn: Perfect Secrecy (informal)</u>

We're no better off determining the plaintext whether or not we see the ciphertext



1. Eve observes everything but c. Guesses m_1
2. Eve observes c. Guesses m_2

Goal: $\Pr[m = m_1] = \Pr[m = m_2]$

Perfect Secrecy [Shannon1945]
(Information Theoretic Secrecy)

<u>Defn Perfect Secrecy (formal):</u>

$$k \xleftarrow{\$} \mathcal{K}$$

$$\forall m_0, m_1 \in \mathcal{M} \quad \text{where} \quad |m_0| = |m_1|$$

$$\forall c \in \mathcal{C}$$

$$\Pr\left[E\left(k, m_0\right) = c\right] = \Pr\left[E\left(k, m_1\right) = c\right]$$

Participation Question

How many OTP keys map m to c?

$$E(k, m) = k \oplus m = c$$

$$D(k, c) = k \oplus c = m$$

A. 1

B. 2

C. Depends on m

Good News: OTP Has Perfect Secrecy

Thm: The one-time pad is perfectly secret

Must show: $$\Pr\left[E(k,m_0)=c\right]=\Pr\left[E(k,m_1)=c\right]$$

where $|M| = |K| = \{0,1\}^m$

Intuition: Say that M = {00,01,10,11}, and *m* = 11. The Adversary receives c = 10. It asks itself whether the plaintext was m_0 or m_1 (e.g., 01 or 10). It reasons:

- if m_0, then $k = m_0 \oplus c = 01 \oplus 10 = 11$.
- if m_1, then $k = m_1 \oplus c = 10 \oplus 10 = 00$.

But all keys are equally likely, so Adv doesn't know which one it is.

Good News: OTP Has Perfect Secrecy

Thm: The one-time pad is perfectly secret

Must show: $$\Pr\left[E(k,m_0)=c\right]=\Pr\left[E(k,m_1)=c\right]$$

where |M| = |K| = $\{0,1\}^m$

Proof:

$$\begin{aligned}
\Pr[E(k,m_0)=c] &= \Pr[k\oplus m_0=c] && (1)\\
&= \frac{|k\in\{0,1\}^m : k\oplus m_0=c|}{\{0,1\}^m} && (2)\\
&= \frac{1}{2^m} && (3)\\
\Pr[E(k,m_1)=c] &= \Pr[k\oplus m_1=c] && (4)\\
&= \frac{|k\in\{0,1\}^m : k\oplus m_1=c|}{\{0,1\}^m} && (5)\\
&= \frac{1}{2^m} && (6)
\end{aligned}$$

Therefore, $\Pr[E(k,m_0)=c]=\Pr[E(k,m_1)=c]$

All Keys Must Be Equally Likely

Two possible messages

Let M={000,001}

Let K = {000, 001, 010, 011, 100, 101, 110, 111}

8 keys

Note that if K is randomly selected as 000, then M = C.

Is this a problem?

Two-time pad is completely broken!

Two Time Pad:

$c_1 = m_1 \oplus k$

$c_2 = m_2 \oplus k$

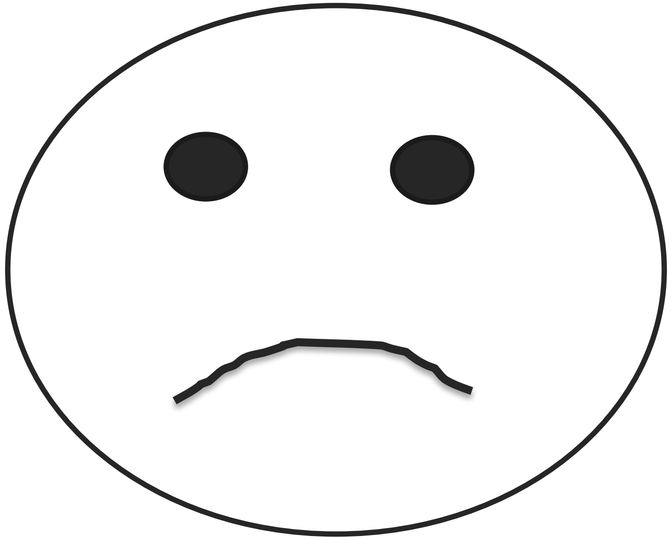
Enough redundancy in ASCII (and English) that $m_1 \oplus m_2$ is enough to know m_1 and m_2

Eavesdropper gets c_1 and c_2
What is the problem?

$c_1 \oplus c_2 = m_1 \oplus m_2$

The “Bad News” Theorem

Theorem: Perfect secrecy requires $|K| >= |M|$



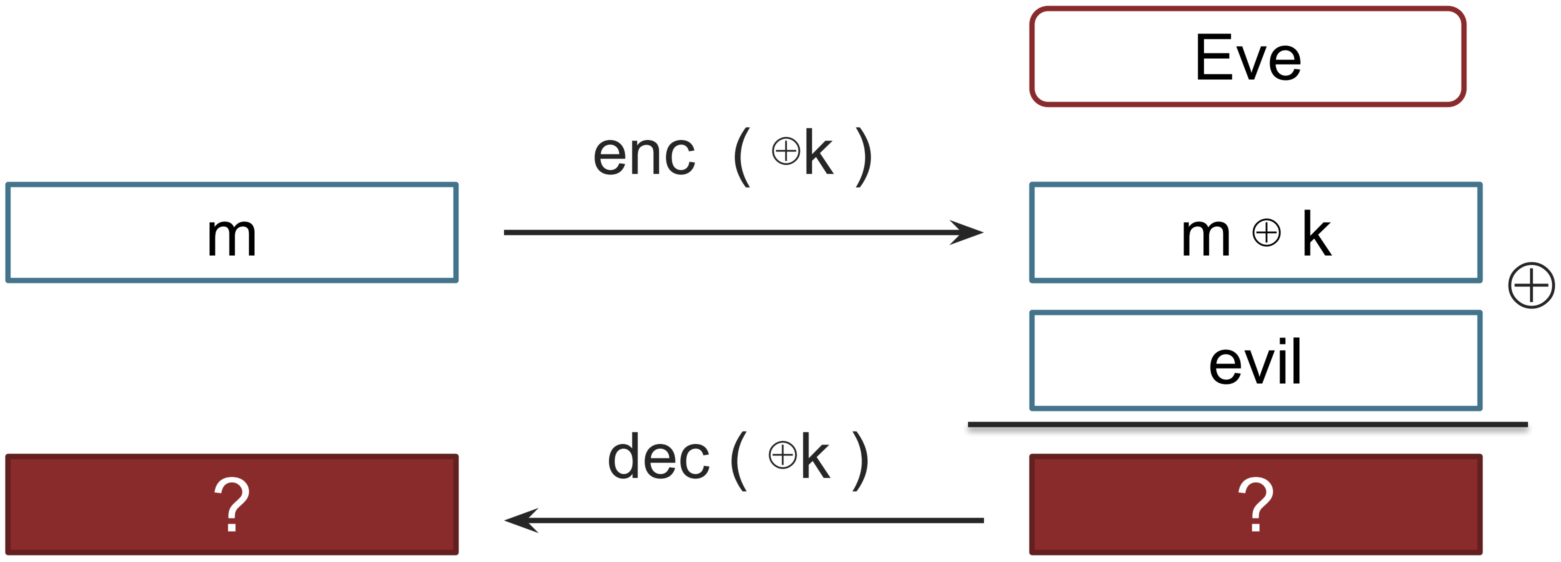
In practice, we usually shoot for
computational security

More bad news

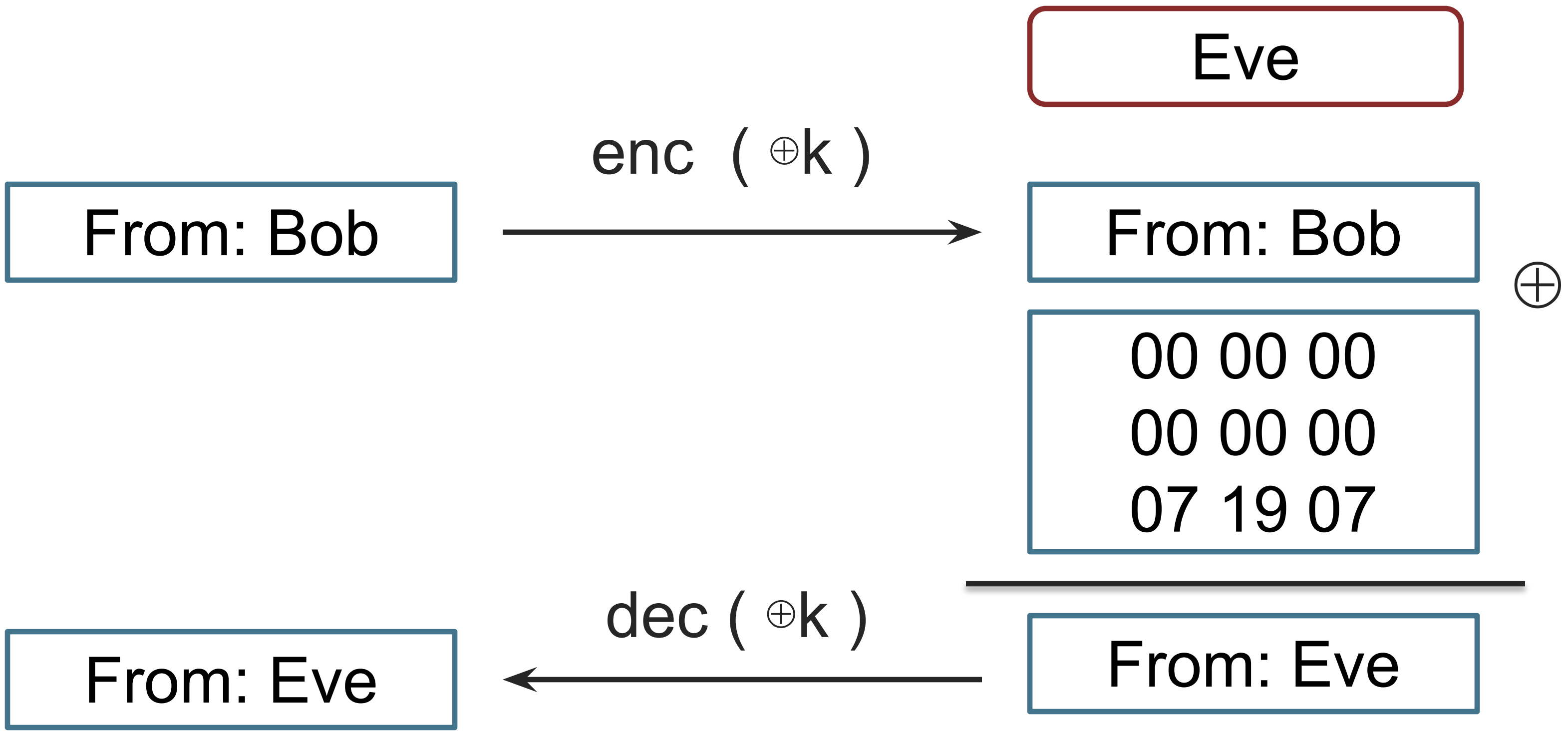
The OTP provides perfect secrecy …

… but is that enough?

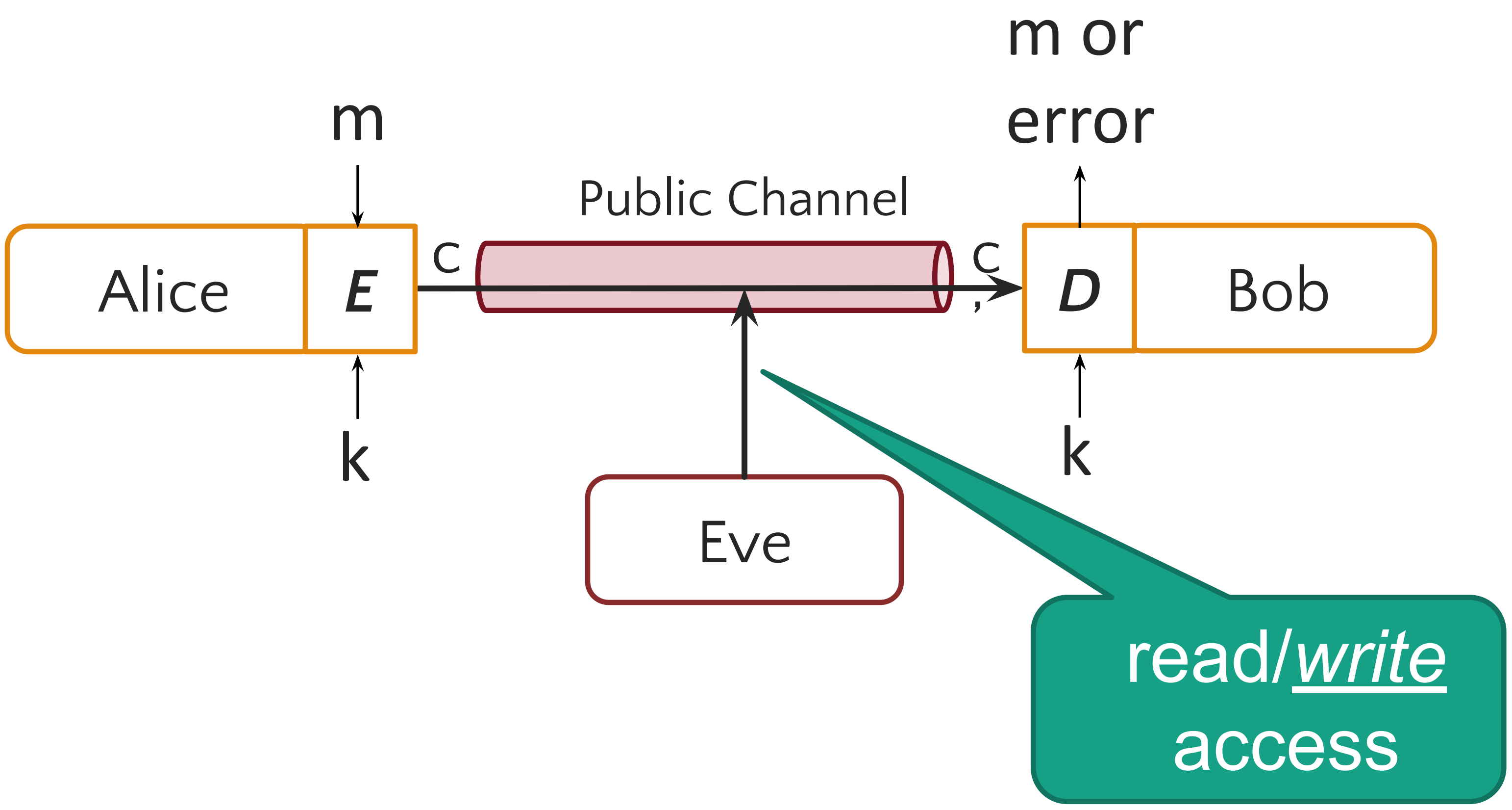
No Integrity



No Integrity



Our Goal: Secure Communication



Sub Goal 2: *Integrity*

Eve should not be able to alter *m* without detection

Ευχαριστώ και καλή μέρα εύχομαι!

Keep hacking!